

TALAT Lecture 2301

Design of Members

Deviation of linear stress distribution

Example 10.1 : Transverse bending of unsymmetrical flange

4 pages

Advanced Level

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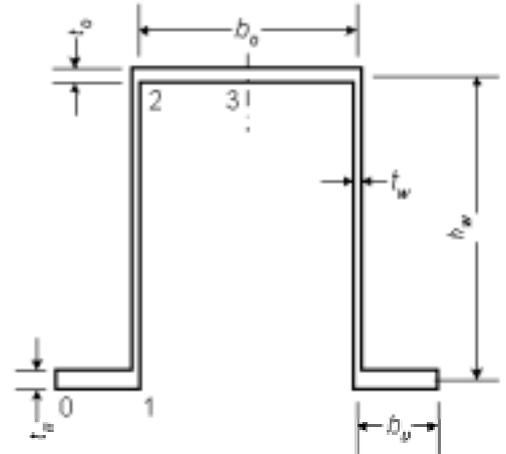
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Example 10.1. Transverse bending of unsymmetrical flange

Check transverse bending of the web due to shear force gradient in the flanges

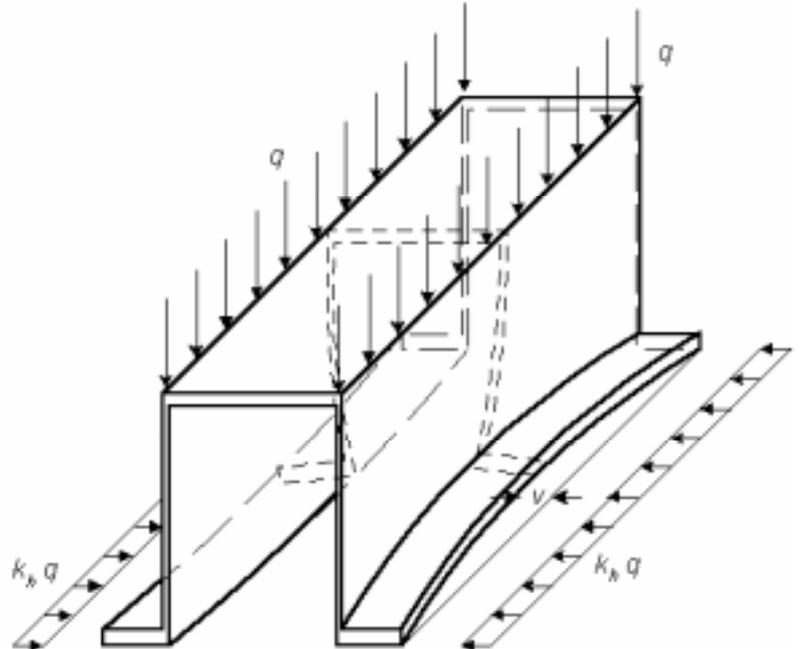
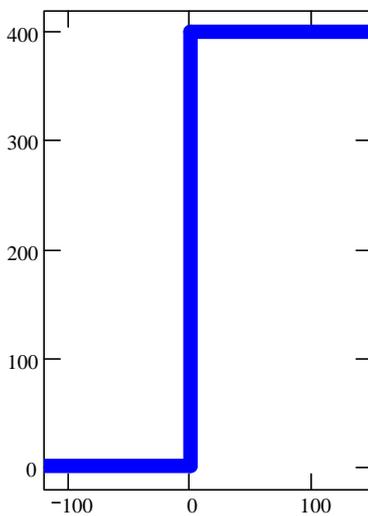
Section height:	$h_w := 400\text{-mm}$	
Flange widths:	$b_o := 300\text{-mm}$	$b_u := 120\text{-mm}$
Flange thickness:	$t_o := 16\text{-mm}$	$t_u := 16\text{-mm}$
Web thickness:	$t_w := 6\text{-mm}$	
Overall length:	$L := 6\text{-m}$	$O := 0\text{-mm}$
Load	$q := 10\text{-kN}\cdot\text{m}^{-1}$	



Cross section, half profile

Co-ordinates

$$y := \begin{bmatrix} -b_u \\ 0 \\ 0 \\ \frac{b_o}{2} \end{bmatrix} \quad z := \begin{bmatrix} 0 \\ 0 \\ h_w \\ h_w \end{bmatrix} \quad t := \begin{bmatrix} t_u \\ t_u \\ t_w \\ t_o \end{bmatrix}$$



$$\begin{aligned} kN &\equiv 1000\text{-newton} \\ MPa &\equiv 10^6\text{-Pa} \\ E &\equiv 70000\text{-MPa} \end{aligned}$$

[1] ENV 1999-1-1. Eurocode 9 - Design of aluminium structures - Part 1-1: General rules. 1997

[2] StBK-N5 Regulations for cold-formed steel and aluminium structures. Svensk Byggtjänst Stockholm 1979

[3] Hetenyi, M. Beams on elastic foundation. University of Michigan, 1958

Nodes	$i := 1.. \text{rows}(y) - 1$	
Area of half cross section	$dA_i := \left[t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]$	$A := \sum_{i=1}^{\text{rows}(y)-1} dA_i$
First moment of area, y axis, gravity centre	$S_y := \sum_{i=1}^{\text{rows}(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$	$z_{gc} := \frac{S_y}{A} \quad z_{gc} = 214.3 \cdot \text{mm}$
Second moment of area, y axis, section modulus	$I_y := \sum_{i=1}^{\text{rows}(y)-1} \left[(z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$	$I_y := I_y - A \cdot z_{gc}^2$
First moment of area, z axis	$S_z := \sum_{i=1}^{\text{rows}(y)-1} (y_i + y_{i-1}) \cdot \frac{dA_i}{2}$	$W_y := \frac{I_y}{z_{gc}} \quad W_y = 9.493 \cdot 10^5 \cdot \text{mm}^3$
Second moment of area with respect to y and z axes	$I_{yz} := \sum_{i=1}^{\text{rows}(y)-1} (2 \cdot y_{i-1} \cdot z_{i-1} + 2 \cdot y_i \cdot z_i + y_{i-1} \cdot z_i + y_i \cdot z_{i-1}) \cdot \frac{dA_i}{6}$	$I_{yz} := I_{yz} - \frac{S_y \cdot S_z}{A} \quad I_{yz} = 5.811 \cdot 10^7 \cdot \text{mm}^4$
Lateral force $k_h \cdot q$ on bottom flange		
Lateral force constant [2]	$k_h := \frac{I_{yz}}{2 \cdot I_y} \quad k_h = 0.143$	
Second moment of area of bottom flange + $0.27 \cdot h_w$ [2]	$A_u := b_u \cdot t_u + 0.27 \cdot h_w \cdot t_w$	$I_{u,z} := \frac{t_u \cdot b_u^3}{3} - \frac{(0.5 \cdot b_u^2 \cdot t_u)^2}{A_u} \quad y_u := \frac{0.5 \cdot b_u^2 \cdot t_u}{A_u}$
Modulus of foundation [3]	$k := \frac{E \cdot t_w^3}{4 \cdot h_w^3} \quad k = 59.062 \cdot \text{kN} \cdot \text{m}^{-2}$	(Transverse bending of top flange neglected)
Parameter λ [3]	$\lambda := \left(\frac{k}{4 \cdot E \cdot I_{u,z}} \right)^{\frac{1}{4}} \quad \lambda = 0.478 \text{ m}^{-1}$	$\lambda \cdot L = 2.867$
Lateral deflection of the bottom flange at the middle of the span [3]	$v_c := \frac{k_h \cdot q}{k} \cdot \left[1 - \frac{2 \cdot \left(\cosh\left(\lambda \cdot \frac{L}{2}\right) \cdot \cos\left(\lambda \cdot \frac{L}{2}\right) \right)}{\cosh(\lambda \cdot L) + \cos(\lambda \cdot L)} \right]$	$v_c = 22.3 \cdot \text{mm}$
Transverse bending moment and stresses in the web	$m_z := k \cdot v_c \cdot h_w \quad m_z = 0.527 \cdot \text{kN}$	$\sigma_{transv} := \frac{m_z \cdot 6}{t_w} \quad \sigma_{transv} = 87.9 \cdot \text{MPa}$
Lateral moment in bottom flange [3]	$M_c := \frac{k_h \cdot q}{\lambda^2} \cdot \frac{\sinh\left(\lambda \cdot \frac{L}{2}\right) \cdot \sin\left(\lambda \cdot \frac{L}{2}\right)}{\cosh(\lambda \cdot L) + \cos(\lambda \cdot L)}$	$M_c = 1.56 \cdot \text{kN} \cdot \text{m} \quad \sigma_c := \frac{M_c \cdot y_u}{I_{u,z}} \quad \sigma_c = 17.3 \cdot \text{MPa}$

Comment: If the web resists $h \cdot q$ by itself then

$$m_z := k \cdot h \cdot q \cdot h_w$$

$$m_z = 0.571 \cdot kN$$

$$v_c := \frac{k \cdot h \cdot q \cdot h_w^3}{3 \cdot E \cdot \frac{t_w^3}{12}}$$

$$v_c = 24.184 \cdot mm$$

$$\sigma_{transv} := \frac{m_z \cdot 6}{t_w^2}$$

$$\sigma_{transv} = 95.2 \cdot MPa$$

Main axis bending

$$M_y := q \cdot \frac{L^2}{8}$$

$$\sigma_x := \frac{M_y}{W_y}$$

$$\sigma_x = 47.4 \cdot MPa$$

$$\sigma_x + \sigma_c = 64.7 \cdot MPa$$