

**TALAT Lecture 2301**  
**Design of Members**  
**Local Buckling**

**Example 3.1 Deflection of class 4 cross section**

5 pages

Advanced Level

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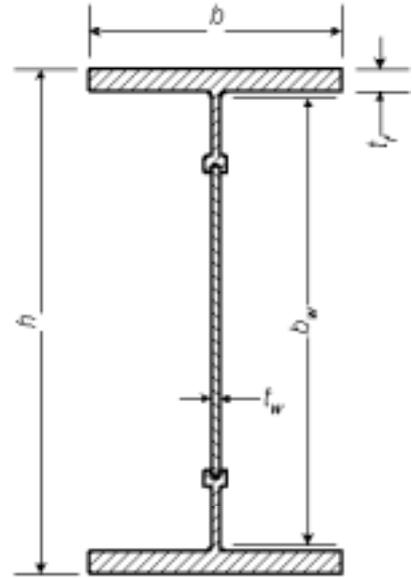
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# Example 3.1 Deflection of class 4 cross section

## Dimensions and material properties

- Total height:  $h := 350 \cdot mm$
- Flange depth:  $b := 110 \cdot mm$
- Flange thickness:  $t_f := 10 \cdot mm$
- Web thickness:  $t_w := 5 \cdot mm$
- Span:  $L := 7.2 \cdot m$
- Width of web plate:  $b_w := h - 2 \cdot t_f$        $b_w = 330 \cdot mm$



[1] Table 3.2b Alloy: **EN AW-6082 T6** EP/O  $t > 5 \text{ mm}$

$f_{0.2} := 260 \cdot MPa$        $E := 70000 \cdot MPa$

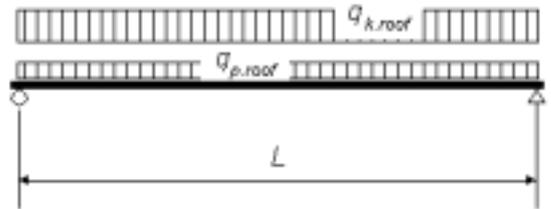
$f_o := f_{0.2}$

## Distributed loads

Characteristic values

Permanent load  $q_{p,roof} := 2.0 \cdot kN \cdot m^{-1}$

Snow load  $q_{k,roof} := 4.0 \cdot kN \cdot m^{-1}$



S.I. units       $kN \equiv 1000 \cdot newton$        $kNm \equiv kN \cdot m$        $MPa \equiv 1000000 \cdot Pa$

## References

- [1] ENV 1999-1-1. Eurocode 9 - Design of aluminium structures - Part 1-1: General rules. 1997
- [2] ENV 1991-1. Eurocode 1 - Basis of design and actions on structures - Part 1: Basis of design. 1994

*Comment: To calculate the deflections a fictitious second moment of area  $I_{fic}$  according to [1] 4.2.4 is used. The second moment of area of the effective cross section in the ultimate limit state  $I_{eff}$  with allowance for local buckling is then needed.*

[1] (4.2) 
$$I_{fic} := I_{gr} - \frac{\sigma}{f_o} \cdot I_{gr} \cdot (I_{gr} - I_{eff})$$

## Local buckling

[1] Tab. 5.1  
Heat treated,  
welded web

a) Web  $g := 0.4$

$$\varepsilon := \sqrt{\frac{250 \cdot \text{MPa}}{f_o}} \quad \varepsilon = 0.981$$

$$\beta_{\bar{w}} = g \cdot \frac{b_w}{t_w} \quad \beta_{\bar{w}} = 26.4$$

$$\beta_{3\bar{w}} = 18 \cdot \varepsilon \quad \beta_{3\bar{w}} = 17.65$$

$$\text{class}_w := \text{if}(\beta_{\bar{w}} > \beta_{3\bar{w}}, 4, 3) \quad \text{class}_w = 4$$

[1] 5.4.5 Reduction factor:

$$\rho_{cw} = \text{if} \left[ \frac{\beta_w}{\varepsilon} \leq 18, 1.0, \frac{29}{\left(\frac{\beta_w}{\varepsilon}\right)} - \frac{198}{\left(\frac{\beta_w}{\varepsilon}\right)^2} \right]$$

$$t_{w.ef} := \text{if}(\text{class}_w \geq 4, t_w \cdot \rho_{cw}, t_w)$$

$$\rho_{cw} = 0.804 \quad t_{w.ef} = 4.0 \cdot \text{mm}$$

b) Flanges

[1] 5.4.3  $\psi := 1$   $g := 1$   
Heat treated  
unwelded flange

$$\beta_f = g \cdot \frac{b - t_w}{2 \cdot t_f} \quad \beta_{\bar{f}} = 5.25$$

$$\beta_{3\bar{f}} = 6 \cdot \varepsilon \quad \beta_{3\bar{f}} = 5.883$$

[1] Tab. 5.1  $\text{class}_f := \text{if}(\beta_{\bar{f}} > \beta_{3\bar{f}}, 4, 3) \quad \text{class}_f = 3$

[1] 5.4.5 Reduction factor:  $\text{class}_f = 3$   $\rho_{cf} = 1$   $t_{f.ef} := t_f$

Classification of the total cross-section:  $\text{class} := \text{if}(\text{class}_f > \text{class}_w, \text{class}_f, \text{class}_w) \quad \text{class} = 4$

### c) Flange induced buckling

[1] 5.12.9 Elastic moment resistance utilized  $k := 0.55$   $f_{of} := f_o$

[1] (5.115)  $LS = \frac{b_w}{t_w} = 66$   $RS = \frac{k \cdot E}{f_{of}} \cdot \sqrt{\frac{b_w \cdot t_w}{b \cdot t_f}} = 181.4$   $LS < RS$  **OK!**

## Bending stiffness

[1] 5.6.2 Elastic modulus of gross cross section  $W_{el}$ :

$$A_{gr} := 2 \cdot b \cdot t_f + (h - 2 \cdot t_f) \cdot t_w \quad A_{gr} = 3.85 \cdot 10^3 \cdot \text{mm}^2$$

$$I_{gr} := \frac{1}{12} \cdot [b \cdot h^3 - (b - t_w) \cdot (h - 2 \cdot t_f)^3] \quad I_{gr} = 7.857 \cdot 10^7 \cdot \text{mm}^4$$

$$W_{el} := \frac{I_{gr} \cdot 2}{h} \quad W_{el} = 4.49 \cdot 10^5 \cdot \text{mm}^3$$

Second moment of area of the effective cross section  $I_{eff}$ :

$$t_f = 10 \cdot mm$$

$$t_{f,ef} = 10 \cdot mm$$

As  $t_{f,ef} = t_f$  then

$$b_c := \frac{b_w}{2} \quad b_c = 165 \cdot mm$$

$$t_w = 5 \cdot mm$$

$$t_{w,ef} = 4 \cdot mm$$

Allowing for local buckling:

$$A_{eff} := A_{gr} - b \cdot (t_f - t_{f,ef}) - b_c \cdot (t_w - t_{w,ef})$$

$$A_{eff} = 3.688 \cdot 10^3 \cdot mm^2$$

Shift of gravity centre:

$$e_{eff} := \left[ b \cdot (t_f - t_{f,ef}) \cdot \left( \frac{h}{2} - \frac{t_f}{2} \right) + \frac{b_c^2}{2} \cdot (t_w - t_{w,ef}) \right] \cdot \frac{1}{A_{eff}}$$

$$e_{eff} = 3.617 \cdot mm$$

Second moment of area with respect to centre of gross cross section:

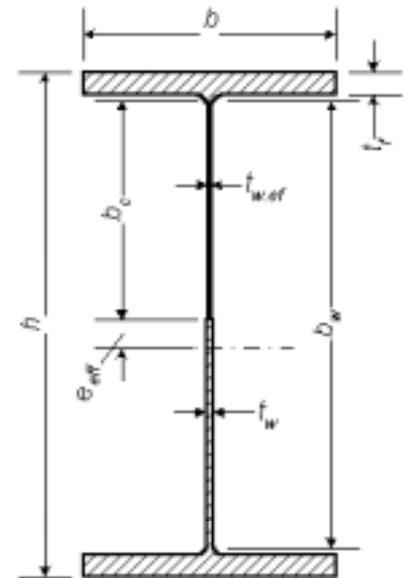
$$I_{eff} := I_{gr} - b \cdot (t_f - t_{f,ef}) \cdot \left( \frac{h}{2} - \frac{t_f}{2} \right)^2 - \frac{b_c^3}{3} \cdot (t_w - t_{w,ef})$$

$$I_{eff} = 7.71 \cdot 10^7 \cdot mm^4$$

Second moment of area with respect to centre of effective cross section:

$$I_{eff} := I_{eff} - e_{eff}^2 \cdot A_{eff}$$

$$I_{eff} = 7.706 \cdot 10^7 \cdot mm^4$$



[1] 4.2.4 (2) Allowing for a reduced stress level  $f_{ic}$  may be used constant along the beam.

[2] 9.5.2 b) Frequent load combination, serviceability limit state, snow load

[2] Table 9.3  $\psi_{II} = 0.2$

$$\psi_{II} = 0.2$$

$$M_{Ed} := (q_{p,roof} + \psi_{II} q_{k,roof}) \cdot \frac{L^2}{8}$$

$$M_{Ed} = 18.1 \cdot kNm$$

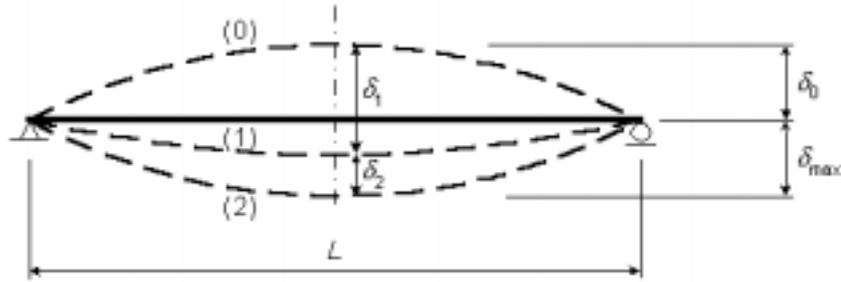
[1] 4.2.4 (2)  $\sigma_{gr} := \frac{M_{Ed}}{W_{el}}$

$$\sigma_{gr} = 40.4 \cdot MPa$$

[1] (4.2)  $I_{fic} := I_{gr} - \frac{\sigma_{gr}}{f_o} \cdot (I_{gr} - I_{eff})$

$$I_{fic} = 7.834 \cdot 10^7 \cdot mm^4$$

## Deflections



$$\delta_i = \frac{5 \cdot q_{p.roof} \cdot L^4}{384 \cdot E \cdot I_{fic}} \quad q_{p.roof} = 2 \cdot \text{kN} \cdot \text{m}^{-1} \quad \delta_i = 12.8 \cdot \text{mm}$$

$$\delta_2 = \psi \cdot \frac{5 \cdot q_{k.roof} \cdot L^4}{384 \cdot E \cdot I_{fic}} \quad q_{k.roof} = 4 \cdot \text{kN} \cdot \text{m}^{-1} \quad \delta_2 = 5.1 \cdot \text{mm}$$

No pre-camber  $\delta_0 = 0 \cdot \text{mm}$

[1] (4.1)  $\delta_{max} = \delta_1 + \delta_2 - \delta_0$   $\delta_{max} = 17.9 \cdot \text{mm}$

[1] 4.2.3  $\delta_{limit} = \frac{L}{360}$  for beams carrying plaster or other brittle finish  $\delta_{limit} = 20 \cdot \text{mm}$

check := if( $\delta_{max} \leq \delta_{limit}$ , "OK!", "Not OK!")  $check = \text{"OK!"}$