

**TALAT Lecture 2301**

**Design of Members**

**Bending Moment**

**Example 4.1 : Bending moment resistance of open  
cross section with closed part**

7 pages

Advanced Level

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# Example 4.1. Bending moment resistance of open cross section with closed part

$f_o := 300 \cdot \text{MPa}$        $\gamma \text{ M}\bar{F} \text{ 1.1}$

Half flange width       $b := 150$

Half section depth       $h := 240$

Half closed part       $c := 75$

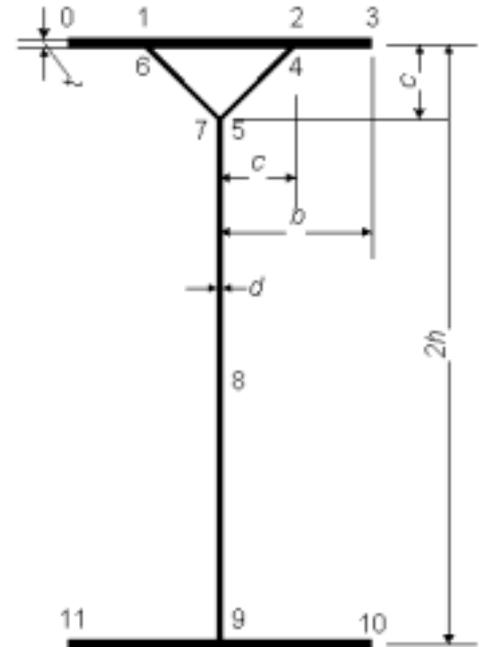
Flange thickness       $t := 8$

Web thickness       $d := 4$

Zero       $o := 0$

- Type of element  
 - Internal element:  $T = 0$   
 - Outstand:  $T > 0$

*Comment: Node 8 is added to indicate shift in neutral axis*

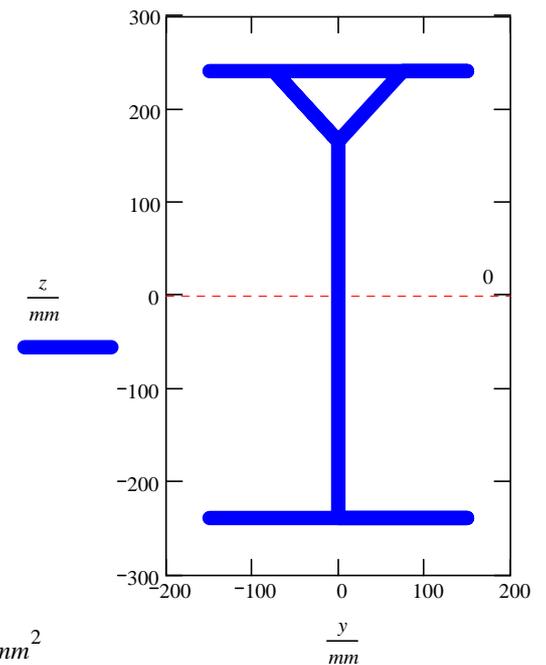


Nodes, co-ordinates (y, z), thickness (t) and types (T)

$i :=$	$y :=$	$z :=$	$t :=$	$T :=$
0	-b	h	o	o
1	-c	h	t	2
2	c	h	t	o
3	b	h	t	1
4	c	h	o	o
5	o	h - c	d	o
6	-c	h	o	o
7	o	h - c	d	o
8	o	o - 1	d	o
9	o	-h	d	o
10	b	-h	o	o
11	-b	-h	t	1

$\text{MPa} \approx 10^6 \cdot \text{Pa}$   
 $\text{kN} \approx 1000 \cdot \text{newton}$

The beam is composed by two extrusions. The weld is close to the neutral axis why HAZ softening does not influence the moment resistance.



Nodes       $i := 1 \dots \text{rows}(y) - 1$        $\text{rows}(y) = 12$

Area of cross section elements  
 $dA_i := \left[ t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]$

Cross section area  
 $A := \sum_{i=1}^{\text{rows}(y)-1} dA_i$        $A = 7.269 \cdot 10^3 \cdot \text{mm}^2$

First moment of area  
Gravity centre

$$S_y := \sum_{i=1}^{\text{rows}(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$$z_{gc} := \frac{S_y}{A}$$

$$z_{gc} = 15.282 \cdot \text{mm}$$

Second moment of area

$$I_y := \sum_{i=1}^{\text{rows}(y)-1} \left[ (z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$$

$$z_{gc,gr} := z_{gc}$$

$$I_{y,gr} := I_y$$

$$I_y := I_y - A \cdot z_{gc}^2$$

$$I_y = 3.344 \cdot 10^8 \cdot \text{mm}^4$$

Elastic section modulus

$$W_{el} := \text{if} \left( \max(z - z_{gc}) > \left| \min(z - z_{gc}) \right|, \frac{I_y}{\max(z - z_{gc})}, \frac{I_y}{\left| \min(z - z_{gc}) \right|} \right)$$

$$W_{el} = 1.31 \cdot 10^6 \cdot \text{mm}^3$$

Torsion constant for open parts

$$I_{tu} := \sum_{i=1}^{\text{rows}(y)-1} dA_i \cdot \frac{(t_i)^2}{3}$$

$$I_{tu} = 1.156 \cdot 10^5 \cdot \text{mm}^4$$

Hollow part in top flange

$$y_h := \begin{bmatrix} y_1 \\ y_2 \\ y_5 \\ y_6 \end{bmatrix} \quad y_h = \begin{bmatrix} -75 \\ 75 \\ 0 \\ -75 \end{bmatrix} \cdot \text{mm} \quad z_h := \begin{bmatrix} z_1 \\ z_2 \\ z_5 \\ z_6 \end{bmatrix} \quad z_h = \begin{bmatrix} 240 \\ 240 \\ 165 \\ 240 \end{bmatrix} \cdot \text{mm}$$

$$t_h := \begin{bmatrix} t_1 \\ t_2 \\ t_5 \\ t_7 \end{bmatrix} \quad t_h = \begin{bmatrix} 8 \\ 8 \\ 4 \\ 4 \end{bmatrix} \cdot \text{mm}$$

Area within mid-line

$$A_{tu} := \sum_{i=1}^{\text{rows}(y_h)-1} 0.5 \cdot (y_{h_i} - y_{h_{i-1}}) \cdot (z_{h_i} + z_{h_{i-1}})$$

$$A_{tu} = 5.625 \cdot 10^3 \cdot \text{mm}^2$$

Sum of l/t

$$Dn := \sum_{i=1}^{\text{rows}(y_h)-1} \text{if} \left[ t_i > 0, \frac{\sqrt{(y_{h_i} - y_{h_{i-1}})^2 + (z_{h_i} - z_{h_{i-1}})^2}}{t_i}, 10^{22} \right]$$

$$Dn = 71.783$$

Torsion constant for closed part

$$I_t := \frac{4 \cdot A_{tu}^2}{Dn}$$

$$I_t = 1.763 \cdot 10^6 \cdot \text{mm}^4$$

Torsion constant whole section

$$I_t := I_t + I_{tu}$$

$$I_t = 1.879 \cdot 10^6 \cdot \text{mm}^4$$

Torsion resistance

$$W_v := 2 \cdot A_{tu} \cdot \min(t_h)$$

$$\min(t_h) = 4 \cdot \text{mm}$$

$$W_v = 4.5 \cdot 10^4 \cdot \text{mm}^3$$

## Effective cross section First iteration

$$5.4.3 (1) \quad \psi_i := \text{if} \left( \left| z_{i-1} - z_{gc} \right| < \left| z_i - z_{gc} \right|, \frac{z_{i-1} - z_{gc}}{z_i - z_{gc}}, \frac{z_i - z_{gc}}{z_{i-1} - z_{gc}} \right) \quad z_{d_i} := \left| z_i - z_{gc} \right|$$

$$\max_z := \max(z_d)$$

$$(5.7 \text{ or } 5.8) \quad g_i := \text{if} \left( \psi_i > -1, 0.7 + 0.30 \cdot \psi_i, \frac{0.8}{1 - \psi_i} \right)$$

Outstand  
Figure 5.2

$$g_i := \text{if}(T_i > 0, 1, g_i)$$

$$5.4.3 (1) \text{ c) } \quad \beta_i := \text{if} \left[ t_i > 0, g_i \cdot \frac{\sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}}{t_i}, 1 \right] \quad \varepsilon_{\bar{o}} = \sqrt{\frac{250 \cdot \text{MPa}}{f_o}} \quad \varepsilon_{\bar{o}} = 0.913$$

$$5.4.4 (5) \quad \varepsilon_i := \text{if} \left[ (z_i \geq z_{gc}) \cdot (z_{i-1} \geq z_{gc}), \text{if} \left( \left| z_{i-1} - z_{gc} \right| < \left| z_i - z_{gc} \right|, \varepsilon_{\bar{o}} \cdot \sqrt{\frac{\max_z}{\left| z_i - z_{gc} \right|}}, \varepsilon_{\bar{o}} \cdot \sqrt{\frac{\max_z}{\left| z_{i-1} - z_{gc} \right|}} \right), 99 \right]$$

$$5.4.5 (3) \text{ a) or c) heat-treated, unwelded} \quad \rho_{\bar{c}} := \text{if} \left[ T_i > 0, \text{if} \left[ \frac{\beta_i}{\varepsilon_i} > 6, 10 \cdot \frac{\varepsilon_i}{\beta_i} - 24 \cdot \left( \frac{\varepsilon_i}{\beta_i} \right)^2, 1.0 \right], \text{if} \left[ \frac{\beta_i}{\varepsilon_i} > 22, 32 \cdot \frac{\varepsilon_i}{\beta_i} - 220 \cdot \left( \frac{\varepsilon_i}{\beta_i} \right)^2, 1.0 \right] \right]$$

Effective thickness

$$t_{eff} := \overrightarrow{\left( \rho_{\bar{c}} \cdot t \right)}$$

$\psi_i =$	$g_i =$	$\beta_i =$	$\varepsilon_i =$	$\frac{\beta_i}{\varepsilon_i} =$	$\rho_{\bar{c}} =$	$\frac{t_{eff_i}}{\text{mm}} =$
1	1	9.375	0.973	9.635	0.779	6.235
1	1	18.75	0.973	19.271	1	8
1	1	9.375	0.973	9.635	0.779	6.235
1	1	1	0.973	1.028	1	0
0.666	0.9	23.862	0.973	24.524	0.939	3.756
0.666	0.9	1	0.973	1.028	1	0
0.666	0.9	23.862	0.973	24.524	0.939	3.756
-0.109	0.667	27.696	99	0.28	1	4
0.064	0.719	42.968	99	0.434	1	4
1	1	1	99	0.01	1	0
1	1	37.5	99	0.379	1	8

*Comment:*  
 $\varepsilon_i$  was given a large value (99)  
for elements entirely on  
the tension side

Area of effective thickness elements

$$dA_i := \overrightarrow{\left[ t_{eff_i} \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]}$$

Cross section area

$$A_{eff} := \sum_{i=1}^{\text{rows}(y)-1} dA_i$$

$$A = 7.269 \cdot 10^3 \cdot \text{mm}^2$$

$$A_{eff} = 6.952 \cdot 10^3 \cdot \text{mm}^2$$

First moment of area

$$S_y := \sum_{i=1}^{\text{rows}(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$$z_{gc} := \frac{S_y}{A_{eff}}$$

Gravity centre

$$z_{gc} = 5.329 \cdot \text{mm}$$

## Effective cross section Second iteration

Node 8 is moved to the neutral axis  
from the first iteration

$$z_8 := z_{gc} + 0.01 \cdot mm$$

$$5.4.3 (1) \quad \psi_i := \text{if} \left( z_{i-1} - z_{gc} < z_i - z_{gc}, \frac{z_{i-1} - z_{gc}}{z_i - z_{gc}}, \frac{z_i - z_{gc}}{z_{i-1} - z_{gc}} \right)$$

$$z_{d_i} := |z_i - z_{gc}|$$

$$\max_z := \max(z_{d_i})$$

$$(5.7 \text{ or } 5.8) \quad g_i := \text{if} \left( \psi_i > -1, 0.7 + 0.30 \cdot \psi_i, \frac{0.8}{1 - \psi_i} \right)$$

$$\max_z = 245.329 \cdot mm$$

Outstand  
element

$$g_i := \text{if}(T_i > 0, 1, g_i)$$

$$5.4.3 (1) c) \quad \beta_i := \text{if} \left[ t_i > 0, g_i \cdot \frac{\sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}}{t_i}, 1 \right]$$

$$z_{gc} = 5.329 \cdot mm$$

$$z_8 = 5.339 \cdot mm$$

$$z_9 = -240 \cdot mm$$

$$5.4.4 (5) \quad \varepsilon_i := \text{if} \left( z_i \geq z_{gc} + (z_{i-1} \geq z_{gc}), \text{if} \left( |z_{i-1} - z_{gc}| < |z_i - z_{gc}|, \varepsilon_i \cdot \sqrt{\frac{\max_z}{|z_i - z_{gc}|}}, \varepsilon_i \cdot \sqrt{\frac{\max_z}{|z_{i-1} - z_{gc}|}} \right), 99 \right)$$

5.4.5 (3)  
a) or c)  
heat-treated,  
unwelded

$$\rho_{\xi} := \text{if} \left[ T_i > 0, \text{if} \left[ \frac{\beta_i}{\varepsilon_i} > 6, 10 \cdot \frac{\varepsilon_i}{\beta_i} - 24 \cdot \left( \frac{\varepsilon_i}{\beta_i} \right)^2, 1.0 \right], \text{if} \left[ \frac{\beta_i}{\varepsilon_i} > 22, 32 \cdot \frac{\varepsilon_i}{\beta_i} - 220 \cdot \left( \frac{\varepsilon_i}{\beta_i} \right)^2, 1.0 \right] \right]$$

Effective  
thickness

$$t_{eff} := \overrightarrow{\rho \cdot t}$$

Max slender-  
ness  
- Internal  
elements

$$\beta_{\xi} := \text{if} \left( T_i > 0, 0, \frac{\beta_i}{\varepsilon_i} \right)$$

$$\beta_{Imax} = \max(\beta_{\xi})$$

$$\beta_{Imax} = 25.686$$

Max slender-  
ness  
- Outstands

$$\beta_{\xi} := \text{if} \left( T_i > 0, \frac{\beta_i}{\varepsilon_i}, 0 \right)$$

$$\beta_{Omax} = \max(\beta_{\xi})$$

$$\beta_{Omax} = 10.044$$

Cross section  
class  
(5.15)

$$\text{class } I := \text{if}(\beta_{Imax} \leq 11, 1, \text{if}(\beta_{Imax} \leq 16, 2, \text{if}(\beta_{Imax} \leq 22, 3, 4)))$$

$$\text{class } I = 4$$

$$\text{class } O := \text{if}(\beta_{Omax} \leq 3, 1, \text{if}(\beta_{Omax} \leq 4.5, 2, \text{if}(\beta_{Omax} \leq 6, 3, 4)))$$

$$\text{class } O = 4$$

$$\text{class} := \text{if}(\text{class } I > \text{class } O, \text{class } I, \text{class } O)$$

$$\text{class} = 4$$

Area of effective  
thickness  
elements

$$dA_i := \overrightarrow{t_{eff_i} \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}}$$

$i =$	$T_i =$	$\psi_i =$	$g_i =$	$\beta_i =$	$\varepsilon_i =$	$\frac{\beta_i}{\varepsilon_i} =$	$\rho_{\xi} =$	$\frac{t_{eff_i}}{mm} =$
1	2		1	9.375	0.933	10.044	0.758	6.062
2	0		1	18.75	0.933	20.088	1	8
3	1		1	9.375	0.933	10.044	0.758	6.062
4	0		1	1	0.933	1.071	1	0
5	0	0.0904	0.904	23.974	0.933	25.686	0.912	3.65
6	0	0.0904	0.904	1	0.933	1.071	1	0
7	0	0.0904	0.904	23.974	0.933	25.686	0.912	3.65
8	0.263	10	0.7	27.941	1.132	24.693	0.935	3.74
9	0	-1.10	0.01	0.613	0.913	0.672	1	4
10	0		1	1	99	0.01	1	0
11	1		1	37.5	99	0.379	1	8

Area of effective cross section

$$A_{eff} := \sum_{i=1}^{rows(y)-1} dA_i$$

$$A = 7.269 \cdot 10^3 \cdot mm^2$$

First moment of area. Gravity centre

$$S_y := \sum_{i=1}^{rows(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$$A_{eff} = 6.862 \cdot 10^3 \cdot mm^2$$

$$z_{gc} := \frac{S_y}{A_{eff}}$$

$$z_{gc} = 3.31 \cdot mm$$

Second moment of area of effective cross section

$$I_y := \sum_{i=1}^{rows(y)-1} \left[ (z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$$

$$I_{eff} := I_y - A_{eff} \cdot z_{gc}^2$$

$$I_{eff} = 3.158 \cdot 10^8 \cdot mm^4$$

Section modulus

$$z_{d_i} := |z_i - z_{gc}| \quad \max_{z,eff} := \max(z_d) \quad W_{eff} := \frac{I_{eff}}{\max_{z,eff}}$$

$$W_{eff} = 1.298 \cdot 10^6 \cdot mm^3$$

Compare gross section

$$z_{d_i} := |z_i - z_{gc,gr}| \quad \max_{z,gr} := \max(z_d) \quad W_{gr} := \frac{I_{y,gr}}{\max_{z,gr}}$$

$$W_{gr} = 1.31 \cdot 10^6 \cdot mm^3$$

## Plastic section modulus

Cross section area of upper part

$$A_{up} := \sum_{i=1}^7 \left[ t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]$$

$$A = 7.269 \cdot 10^3 \cdot mm^2$$

$$A_{up} = 3.249 \cdot 10^3 \cdot mm^2$$

Move node 8 to plastic neutral axis

$$z_8 := z_7 - \frac{\frac{A}{2} - A_{up}}{t_8}$$

$$z_8 = 68.566 \cdot mm$$

Plastic section modulus

$$W_{pl} := \sum_{i=1}^{rows(y)-1} \frac{|z_i + z_{i-1}|}{2} \cdot \left[ t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]$$

$$W_{pl} = 1.475 \cdot 10^6 \cdot mm^3$$

$$\frac{W_{pl}}{W_{el}} = 1.126$$

## Shape factor

$$(5.15) \quad \alpha_{3I} = 1 + \frac{22 - \beta}{22 - 16} \cdot \frac{I_{max}}{I_{el}} \cdot \left( \frac{W_{pl}}{W_{el}} - 1 \right) \quad \alpha_{3O} = 1 + \frac{6 - \beta}{6 - 4.5} \cdot \left( \frac{W_{pl}}{W_{el}} - 1 \right)$$

$$\alpha_{3I} = \text{if}(\alpha_{3I} < \alpha_{3O}, \alpha_{3I}, \alpha_{3O}) \quad \alpha_{3I} = 0.661$$

$$\alpha := \text{if} \left( class < 3, \frac{W_{pl}}{W_{el}}, \text{if} \left( class < 4, \alpha_{3I}, \frac{W_{eff}}{W_{el}} \right) \right) \quad class = 4 \quad \alpha = 0.991$$

## Bending moment resistance

$$(5.14) \quad M_{Rd} := \alpha \cdot W_{el} \frac{f_o}{\gamma_{MI}}$$

$$M_{Rd} = 354 \cdot kN \cdot m$$

Cross section class = 4

Elastic neutral axis:

- first iteration dashed-dotted line
- second iteration dashed line

Plastic neutral axis dotted line

