TALAT Lecture 2301

Design of Members

Bending Moment

Example 4.2 : Hollow cross section (polygon)

5 pages

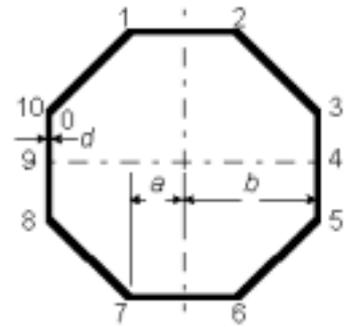Advanced Level

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Example 4.2 Hollow cross section (polygon)

Half width $\quad b := 50 \cdot mm$

Thickness $\quad s := 1.2 \cdot mm$

Half width of flat parts $\quad a := \dfrac{b}{1 + \sqrt{2}}$

$O := 0 \cdot mm \qquad a = 20.711 \cdot mm$

Co-ordinates and thickness

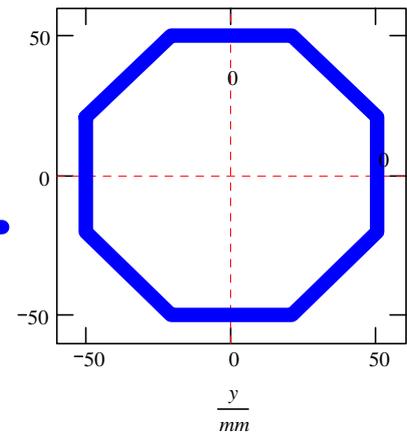*Comment: Nodes 4 and 9 are added to indicate shift in neutral axis*

$$y := \begin{bmatrix} -b \\ -a \\ a \\ b \\ b \\ b \\ a \\ -a \\ -b \\ -b \\ -b \end{bmatrix} \qquad z := \begin{bmatrix} a \\ b \\ b \\ a \\ O \\ -a \\ -b \\ -b \\ -a \\ O \\ a \end{bmatrix} \qquad t := \begin{bmatrix} s \\ s \\ s \\ s \\ s \\ s \\ s \\ s \\ s \\ s \\ s \end{bmatrix}$$

$f_o := 300 \cdot MPa \qquad MPa \equiv 10^6 \cdot Pa$

$\gamma_{M1} := 1.1 \qquad kN \equiv 1000 \cdot newton$

Nodes $\qquad i := 1 .. \, rows(y) - 1$

Gross cross section

Area of cross section elements
$$dA_i := \left[t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]$$

Cross section area
$$A := \sum_{i=1}^{rows(y)-1} dA_i$$

$A = 397.6 \cdot mm^2$

First moment of area. Gravity centre
$$S_y := \sum_{i=1}^{rows(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$z_{gc} := \dfrac{S_y}{A}$

$z_{gc} = 0 \cdot mm$

$z_{gc.gr} := z_{gc}$

Second moment of area
$$I_y := \sum_{i=1}^{rows(y)-1} \left[(z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$$

$I_y := I_y - A \cdot z_{gc}^2$

$I_y = 5.255 \cdot 10^5 \cdot mm^4$

Area within mid-line
$$A_v := \sum_{i=1}^{rows(y)-1} 0.5 \cdot (y_i - y_{i-1}) \cdot (z_i + z_{i-1})$$

$A_v = 8.284 \cdot 10^3 \cdot mm^2$

Check:

$(2 \cdot b)^2 - 2 \cdot (b - a)^2 = 8.284 \cdot 10^3 \cdot mm^2$

Sum of l/t
$$Dn := \sum_{i=1}^{rows(y)-1} \frac{\sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}}{t_i}$$

$Dn = 276.142$

Torsion constant
$$I_v := \frac{4 \cdot A_v^2}{Dn}$$

$I_v = 9.941 \cdot 10^5 \cdot mm^4$

Torsion resistance
$W_v := 2 \cdot A_v \cdot min(t) \qquad min(t) = 1.2 \cdot mm$

$W_v = 1.988 \cdot 10^4 \cdot mm^3$

Effective cross section First iteration

Zero position $z_d := 0$ $\rho_c = 0$

5.4.3 (1)

$$\psi_i := if\left(\left|z_{i-1} - z_{gc}\right| < \left|z_i - z_{gc}\right|, \frac{z_{i-1} - z_{gc}}{z_i - z_{gc}}, \frac{z_i - z_{gc}}{z_{i-1} - z_{gc}}\right)$$

$z_{d_i} := \left|z_i - z_{gc}\right|$

$max_z := max\left(z_d\right)$

(5.7 or 5.8)

$$g_i := if\left(\psi_i > -1, 0.7 + 0.30 \cdot \psi_i, \frac{0.8}{1 - \psi_i}\right)$$

$max_z = 50 \cdot mm$

5.4.3 (1) c)

$$\beta_i := if\left[t_i > 0, g_i \cdot \frac{\sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}}{t_i}, 1\right]$$

$\varepsilon_o := \sqrt{\frac{250 \cdot MPa}{f_o}}$ $\varepsilon_o = 0.913$

5.4.4 (5)

$$\varepsilon_i := if\left[(z_i > z_{gc}) + (z_{i-1} > z_{gc}), if\left(\left|z_{i-1} - z_{gc}\right| < \left|z_i - z_{gc}\right|, \varepsilon_o \cdot \sqrt{\frac{max_z}{\left|z_i - z_{gc}\right|}}, \varepsilon_o \cdot \sqrt{\frac{max_z}{\left|z_{i-1} - z_{gc}\right|}}\right), 99\right]$$

5.4.5 (3) c)
heat-treated,
unwelded

$$\rho_{c_i} := if\left[\frac{\beta_i}{\varepsilon_i} > 22, 32 \cdot \frac{\varepsilon_i}{\beta_i} - 220 \cdot \left(\frac{\varepsilon_i}{\beta_i}\right)^2, 1.0\right]$$

Comment: ε_i is given a large value (99)
for elements entirely on the tension side

Effective
thickness

$$t_{eff} := \overrightarrow{(\rho_c \cdot c)}$$

$\psi_i =$	$g_i =$	$\beta_i =$	$\varepsilon_i =$	$\dfrac{\beta_i}{\varepsilon_i} =$	$\rho_{c} =$	$\dfrac{t_{eff_i}}{mm} =$
0.414	0.824	28.452	0.913	31.167	0.8	0.96
1	1	34.518	0.913	37.812	0.692	0.831
0.414	0.824	28.452	0.913	31.167	0.8	0.96
0	0.7	12.081	1.418	8.518	1	1.2
0	0.7	12.081	99	0.122	1	1.2
0.414	0.824	28.452	99	0.287	1	1.2
1	1	34.518	99	0.349	1	1.2
0.414	0.824	28.452	99	0.287	1	1.2
0	0.7	12.081	99	0.122	1	1.2
0	0.7	12.081	1.418	8.518	1	1.2

Area of effective
thickness
elements

$$dA_i := \left[t_{eff_i} \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}\right]$$

Cross
section
area

$$A_{eff} := \sum_{i=1}^{rows(y)-1} dA_i$$

$A_{eff} = 362.498 \cdot mm^2$ $A = 397.645 \cdot mm^2$

First
moment
of area.

$$S_y := \sum_{i=1}^{rows(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$$z_{gc} := \frac{S_y}{A_{eff}}$$

Gravity centre

$z_{gc} = -4.046 \cdot mm$

The nodes in the vertical webs are moved to
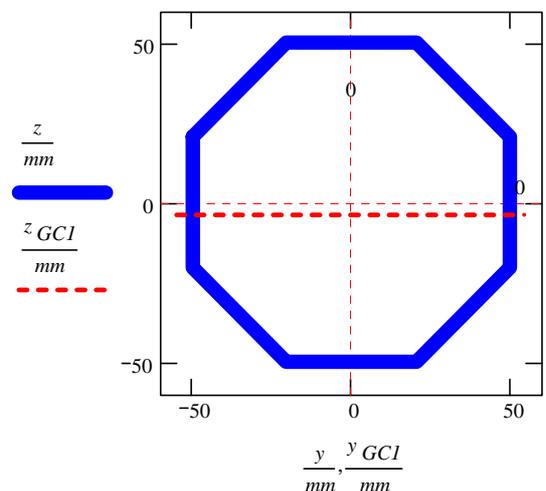the neutral axis from first iteration

$i := 0 .. rows(y) - 2$

$$z_i := if\left[(z_i > z_{gc}) \cdot (z_{i+1} < z_{gc}), z_{gc}, z_i\right]$$

$i := 1 .. rows(y) - 1$

$$z_i := if\left[(z_i > z_{gc}) \cdot (z_{i-1} < z_{gc}), z_{gc}, z_i\right]$$

Check $z_4 = -4.046 \cdot mm$ $z_9 = -4.046 \cdot mm$ OK !

$\dfrac{z}{mm}$

$\dfrac{z_{GC1}}{mm}$

$\dfrac{y}{mm}, \dfrac{y_{GC1}}{mm}$

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Second iteration

5.4.3 (1)
$$\psi_i := if\left(\left|z_{i-1} - z_{gc}\right| < \left|z_i - z_{gc}\right|, \frac{z_{i-1} - z_{gc}}{z_i - z_{gc}}, \frac{z_i - z_{gc}}{z_{i-1} - z_{gc}}\right)$$

$z_{gc} = -4.046 \cdot mm$

$z_{d_i} := \left|z_i - z_{gc}\right|$

(5.7 or 5.8)
$$g_i := if\left(\psi_i > -1, 0.7 + 0.30 \cdot \psi_i, \frac{0.8}{1 - \psi_i}\right)$$

$max_z := max\left(z_d\right)$

$max_z = 54.046 \cdot mm$

5.4.3 (1) c)
$$\beta_i := if\left[t_i > 0, g_i \cdot \frac{\sqrt{\left(y_i - y_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2}}{t_i}, 1\right]$$

5.4.4 (5)
$$\varepsilon_i := if\left[\left(z_i > z_{gc}\right) + \left(z_{i-1} > z_{gc}\right), if\left(\left|z_{i-1} - z_{gc}\right| < \left|z_i - z_{gc}\right|, \varepsilon_o \cdot \sqrt{\frac{max_z}{\left|z_i - z_{gc}\right|}}, \varepsilon_o \cdot \sqrt{\frac{max_z}{\left|z_{i-1} - z_{gc}\right|}}\right), 99\right]$$

5.4.5 (3) c)
heat-treated,
unwelded
$$\rho_{c_i} := if\left[\frac{\beta_i}{\varepsilon_i} > 22, 32 \cdot \frac{\varepsilon_i}{\beta_i} - 220 \cdot \left(\frac{\varepsilon_i}{\beta_i}\right)^2, 1.0\right]$$

Effective
thickness
$$t_{eff} := \overrightarrow{\left(\rho_c \cdot t\right)}$$

Max slender-
ness
- Internal
elements
$$\beta_{i\_e} := \frac{\beta_i}{\varepsilon_i} \qquad \beta_{max} := max\left(\beta_e\right)$$

$$\beta_{max} = 37.812$$

Cross section
class
$$class := if\left(\beta_{max} \le 22, 3, 4\right) \qquad class = 4$$

Area of
effective
thickness
elements
$$dA_i := \overrightarrow{\left[t_{eff_i} \cdot \sqrt{\left(y_i - y_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2}\right]}$$

$\psi_i =$	$g_i =$	$\beta_i =$	$\varepsilon_i =$	$\frac{\beta_i}{\varepsilon_i} =$	$\rho_{c_i} =$	$\frac{t_{eff_i}}{mm} =$
0.458	0.837	28.906	0.913	31.665	0.791	0.949
1	1	34.518	0.913	37.812	0.692	0.831
0.458	0.837	28.906	0.913	31.665	0.791	0.949
0	0.7	14.441	1.349	10.707	1	1.2
0	0.7	9.721	99	0.098	1	1.2
0.363	0.809	27.918	99	0.282	1	1.2
1	1	34.518	99	0.349	1	1.2
0.363	0.809	27.918	99	0.282	1	1.2
0	0.7	9.721	99	0.098	1	1.2
0	0.7	14.441	1.349	10.707	1	1.2

Cross
section
area
$$A_{eff} := \sum_{i=1}^{rows(y)-1} dA_i$$

$$A_{eff} = 361.596 \cdot mm^2$$

First
moment
of area.
$$S_y := \sum_{i=1}^{rows(y)-1} \left(z_i + z_{i-1}\right) \cdot \frac{dA_i}{2}$$

$$z_{gc} := \frac{S_y}{A_{eff}}$$

Gravity centre
$$z_{gc} = -4.144 \cdot mm$$

Gross cross
section
$$I_{y.gr} := I_y$$

$$I_{y.gr} = 5.255 \cdot 10^5 \cdot mm^4$$

$\frac{z}{mm}$

$\frac{z_{GC1}}{mm}$

$\frac{z_{GC2}}{mm}$

$\frac{y}{mm}, \frac{y_{GC1}}{mm}$
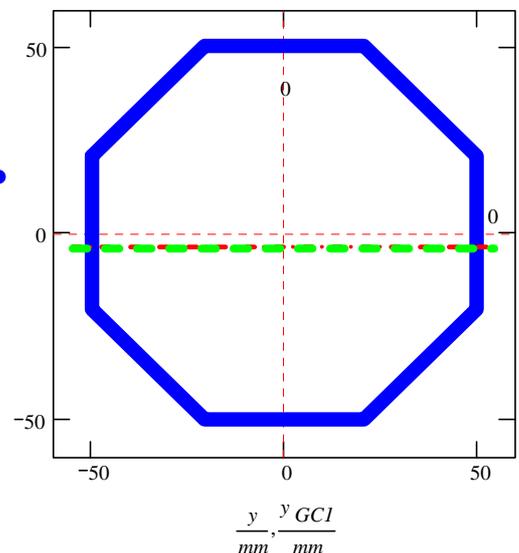
| | Second moment of area of effective cross section | $I_y := \sum_{i=1}^{rows(y)-1} \left[(z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$ | $I_{eff} := I_y - A \cdot z_{gc}^2$ |

$$I_{eff} = 5.187 \cdot 10^5 \circ mm^4$$

Section modulus $z_{d_i} := \left| z_i - z_{gc} \right|$ $max_{z.eff} := max(z_d)$ $W_{eff} := \dfrac{I_{eff}}{max_{z.eff}}$ $W_{eff} = 9.579 \cdot 10^3 \circ mm^3$

Compare gross section $z_{d_i} := \left| z_i - z_{gc.gr} \right|$ $max_{z.gr} := max(z_d)$ $W_{gr} := \dfrac{I_{y.gr}}{max_{z.gr}}$ $W_{gr} = 1.051 \cdot 10^4 \circ mm^3$

Moment resistance $M_{Rd} := W_{eff} \dfrac{f_o}{\gamma_{M1}}$ $M_{Rd} = 2.613 \circ kN \cdot m$

Axial force

Move node close to GC1 to nearest node $z_i := if\left(\left| z_i - z_{GC1} \right| < 1 \cdot mm, z_{i-1}, z_i \right)$

5.4.3 (1) c) $\beta_i := if\left[t_i > 0, \dfrac{\sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}}{t_i}, 1 \right]$

5.4.5 (3) c) heat-treated, unwelded $\rho_{c_i} := if\left[\dfrac{\beta_i}{\varepsilon_o} > 22, 32 \cdot \dfrac{\varepsilon_o}{\beta_i} - 220 \cdot \left(\dfrac{\varepsilon_o}{\beta_i} \right)^2, 1.0 \right]$ $\varepsilon_{\bar{o}} 0.913$

Effective thickness $t_{eff} := \overrightarrow{(\rho_c \cdot t)}$

$i =$	$\beta_i =$	$\dfrac{\beta_i}{\varepsilon_o} =$	$\rho_{c} =$	$\dfrac{t_{eff_i}}{mm} =$
1	34.518	37.812	0.692	0.831
2	34.518	37.812	0.692	0.831
3	34.518	37.812	0.692	0.831
4	0	0	1	1.2
5	34.518	37.812	0.692	0.831
6	34.518	37.812	0.692	0.831
7	34.518	37.812	0.692	0.831
8	34.518	37.812	0.692	0.831
9	0	0	1	1.2
10	34.518	37.812	0.692	0.831

Area of effective thickness elements $dA_i := \overrightarrow{\left[t_{eff_i} \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]}$

Cross section area $A_{eff} := \sum_{i=1}^{rows(y)-1} dA_i$ $A_{eff} = 275.335 \circ mm^2$

First moment of area. Gravity centre $S_y := \sum_{i=1}^{rows(y)-1} (z_i + z_{i-1}) \cdot \dfrac{dA_i}{2}$ $z_{gc} := \dfrac{S_y}{A_{eff}}$

$$z_{gc} = 0 \circ mm$$

Axial force resistance $N_{Rd} := A_{eff} \cdot \dfrac{f_o}{\gamma_{M1}}$

$$N_{Rd} = 75.091 \circ kN$$

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