TALAT Lecture 2301

Design of Members

Bending Moment

Example 4.3 : Bending moment resistance of welded hollow section with outstands. Class 2 cross section

10 pages

Advanced Level

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Example 4.3. Bending moment resistance of welded hollow section with outstands. Class 2 cross section

Half top flange $a := 150 \cdot mm$

Half bottom flange $b := 66 \cdot mm$

Section dept $h := 99 \cdot mm$

Flange thickness $d := 15 \cdot mm$

Web thickness $w := 18 \cdot mm$

Bottom flange t. $t := 18 \cdot mm$

Type of element
- Internal element: $T = 0$
- Outstand: $T > 0$

$o := 0 \cdot mm$

Half hollow width $c := 96 \cdot mm$

Width of HAZ $haz := 30 \cdot mm$

HAZ reduction $\rho_{haz} := 0.8$

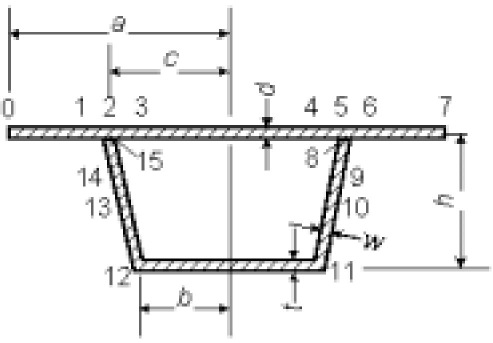
$$bz := c - \frac{haz}{\sqrt{(c-b)^2 + h^2}} \cdot (c - b)$$

$$hz := \frac{haz}{\sqrt{(c-b)^2 + h^2}} \cdot h$$

$f_o := 200 \cdot MPa$

$\gamma_{M1} := 1.1$

$MPa \equiv 10^6 \cdot Pa$



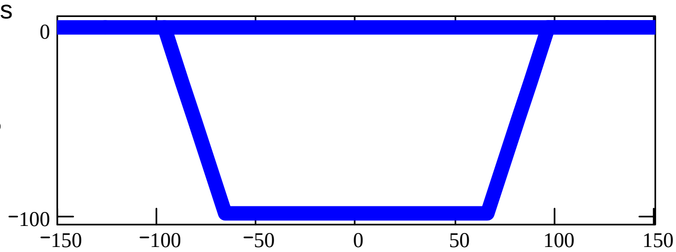
Nodes, co-ordinates (y, z), thickness (t) and types (T)

$$i := \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{bmatrix} \quad y := \begin{bmatrix} -a \\ -c - haz \\ -c \\ -c + haz \\ c - haz \\ c \\ c + haz \\ a \\ c \\ bz \\ 0.5 \cdot (c + b) \\ b \\ -b \\ -0.5 \cdot (c + b) \\ -bz \\ -c \end{bmatrix} \quad z := \begin{bmatrix} o \\ o \\ o \\ o \\ o \\ o \\ o \\ o \\ o \\ -hz \\ -0.5 \cdot h \\ -h \\ -h \\ -0.5 \cdot h \\ -hz \\ o \end{bmatrix} \quad t := \begin{bmatrix} o \\ d \\ d \\ d \\ d \\ d \\ d \\ d \\ o \\ w \\ w \\ w \\ t \\ w \\ w \\ w \end{bmatrix} \quad T := \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Comments:
Node 10 and 13 indicate shift in neutral axis.
Element 1-2, 2-3, 4-5, 5-6, 8-9 and 14-15 define HAZs.

This solution of the example is comprehensive because:
1. Calculations are shown in every detail
2. It covers all classes of cross section
3. The possibility to increase the effective thickness for elements not fully stressed is shown. The iteration procedure is then mandatory. However, in this example, the result is almost exactly the same as if a simplified method was used, that's because there is no reduction of the web thickness due to local buckling.

The beam is composed by two extrusions. The weld is close to the neutral axis why HAZ softening does not influence the moment resistance.



Nodes $i := 1 .. rows(y) - 1 \qquad rows(y) = 16$

Area of cross section elements

$$dA_i := \overrightarrow{\left[t_i \cdot \sqrt{\left(y_i - y_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2} \right]}$$

Cross section area

$$A := \sum_{i=1}^{rows(y)-1} dA_i$$

$A = 1.06 \cdot 10^4 \, mm^2$

First moment of area Gravity centre

$$S_y := \sum_{i=1}^{rows(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$$z_{gc} := \frac{S_y}{A}$$

$$z_{gc} = -39.581 \circ mm$$

$$z_{gc.gr} := z_{gc}$$

Second moment of area

$$I_y := \sum_{i=1}^{rows(y)-1} \left[(z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$$

$$I_y := I_y - A \cdot z_{gc}^2$$

$$I_{y.gr} := I_y \qquad I_y = 1.885 \cdot 10^7 \circ mm^4$$

Elastic section modulus

$$W_{el} := if\left(max(z - z_{gc}) > \left| min(z - z_{gc}) \right|, \frac{I_y}{max(z - z_{gc})}, \frac{I_y}{\left| min(z - z_{gc}) \right|} \right)$$

$$W_{el} = 3.172 \cdot 10^5 \circ mm^3$$

Torsion constant for open parts

$$I_{tu} := \sum_{i=1}^{rows(y)-1} dA_i \cdot \frac{(t_i)^2}{3}$$

$$I_{tu} = 9.963 \cdot 10^5 \circ mm^4$$

Hollow part, (nodes 2, 5, 11, 12 and 15)

$$y_h := \begin{bmatrix} y_2 \\ y_5 \\ y_{11} \\ y_{12} \\ y_{15} \end{bmatrix} \quad y_h = \begin{bmatrix} -96 \\ 96 \\ 66 \\ -66 \\ -96 \end{bmatrix} \circ mm \quad z_h := \begin{bmatrix} z_2 \\ z_5 \\ z_{11} \\ z_{12} \\ z_{15} \end{bmatrix} \quad z_h = \begin{bmatrix} 0 \\ 0 \\ -99 \\ -99 \\ 0 \end{bmatrix} \circ mm \quad t_h := \begin{bmatrix} t_1 \\ t_4 \\ t_{11} \\ t_{12} \\ t_{14} \end{bmatrix} \quad t_h = \begin{bmatrix} 15 \\ 15 \\ 18 \\ 18 \\ 18 \end{bmatrix} \circ mm$$

Area within mid-line

$$A_{tu} := \sum_{i=1}^{rows(y_h)-1} 0.5 \cdot \left(y_{h_i} - y_{h_{i-1}} \right) \cdot \left(z_{h_i} + z_{h_{i-1}} \right)$$

$$A_{tu} = 1.604 \cdot 10^4 \circ mm^2$$

Sum of l/t

$$Dn := \sum_{i=1}^{rows(y_h)-1} if\left[t_i > 0, \frac{\sqrt{\left(y_{h_i} - y_{h_{i-1}} \right)^2 + \left(z_{h_i} - z_{h_{i-1}} \right)^2}}{t_{h_i}}, 10^{22} \right]$$

$$Dn = 31.627$$

Torsion constant for closed part

$$I_t := \frac{4 \cdot A_{tu}^2}{Dn}$$

$$I_t = 3.253 \cdot 10^7 \circ mm^4$$

Torsion constant whole section

$$I_t := I_t + I_{tu}$$

$$I_t = 3.353 \cdot 10^7 \circ mm^4$$

Torsion resistance

$$W_v := 2 \cdot A_{tu} \cdot min(t_h) \qquad min(t_h) = 15 \circ mm$$

$$W_v = 4.811 \cdot 10^5 \circ mm^3$$

$$z_{gc} = -39.581 \circ mm$$

Effective cross section. First iteration

5.4.3 (1) $\psi_i := 1 \quad j := 9..11 \quad \psi_j := \frac{z_8 - z_{gc}}{z_{11} - z_{gc}} \quad \psi_{j+4} := \psi_j$

$z_{d_i} := |z_i - z_{gc}|$

$max_z := max(z_d)$

(5.7 or 5.8) $g_i := if\left(\psi_i > -1, 0.7 + 0.30\cdot\psi_i, \frac{0.8}{1 - \psi_i}\right)$

Outstand Figure 5.2 $g_i := if(T_i > 0, 1, g_i)$

5.4.3 (1) c) $k := 1..2 \quad \beta_k := \frac{y_2 - y_0 - 0.5\cdot t_{13}}{t_2} \quad \beta_{k+5} := \beta_k$

$l := 3..5 \quad \beta_l := \frac{y_5 - y_2 - t_{13}}{t_4}$

$\beta_j := if\left[t_{11} > 0, g_j\cdot\frac{\sqrt{(y_{11} - y_8)^2 + (z_{11} - z_8)^2}}{t_{11}}, 1\right] \quad \beta_{j+4} := \beta_j$

5.4.4 (5) $\varepsilon_o := \sqrt{\frac{250\cdot MPa}{f_o}} \quad \varepsilon_i := \varepsilon_o\cdot\sqrt{\frac{max_z}{|z_{gc}|}} \quad \varepsilon_i := if\left[(z_i \ge z_{gc})\cdot(z_{i-1} \ge z_{gc}), \varepsilon_i, 99\right] \quad \varepsilon_o = 1.118$

5.4.5 (3) a) or c) non heat treated, welded

$\rho_{c_i} := if\left[T_i > 0, if\left[\frac{\beta_i}{\varepsilon_i} > 4, 8\cdot\frac{\varepsilon_i}{\beta_i} - 16\cdot\left(\frac{\varepsilon_i}{\beta_i}\right)^2, 1.0\right], if\left[\frac{\beta_i}{\varepsilon_i} > 15, 25\cdot\frac{\varepsilon_i}{\beta_i} - 150\cdot\left(\frac{\varepsilon_i}{\beta_i}\right)^2, 1.0\right]\right]$

Effective thickness $t_{eff} := \overrightarrow{(\rho_c\cdot t)}$

Heat Affected Zone

Flange $t_{haz.f} := \rho_{haz} \cdot t_2$ $t_{haz.f} = 12 \cdot mm$

Web $t_{haz.w} := \rho_{haz} \cdot t_9$ $t_{haz.w} = 14.4 \cdot mm$

$n := 2 .. 3$

$t_{eff_n} := if\left(t_{eff_n} > t_{haz.f}, t_{haz.f}, t_{eff_n}\right)$ $t_{eff_{8-n}} := t_{eff_n}$

$t_{eff_9} := if\left(t_{eff_9} > t_{haz.w}, t_{haz.w}, t_{eff_9}\right)$ $t_{eff_{15}} := t_{eff_9}$

Comment:
ε_i was given a large value (99) for elements entirely on the tension side

$\psi_i =$	$g_i =$	$\beta_i =$	$\varepsilon_i =$	$\frac{\beta_i}{\varepsilon_i} =$	$\rho_{c_i} =$	$\frac{t_{eff_i}}{mm} =$
1	1	3	1.37	2.19	1	15
1	1	3	1.37	2.19	1	12
1	1	11.6	1.37	8.468	1	12
1	1	11.6	1.37	8.468	1	15
1	1	11.6	1.37	8.468	1	12
1	1	3	1.37	2.19	1	12
1	1	3	1.37	2.19	1	15
1	1	0	1.37	0	1	0
-0.666	0.5	2.874	1.37	2.098	1	14.4
-0.666	0.5	2.874	99	0.029	1	18
-0.666	0.5	2.874	99	0.029	1	18
1	1	0	99	0	1	18
-0.666	0.5	2.874	99	0.029	1	18
-0.666	0.5	2.874	99	0.029	1	18
-0.666	0.5	2.874	1.37	2.098	1	14.4

Area of effective thickness elements

$$dA_i := \overrightarrow{\left[t_{eff_i} \cdot \sqrt{\left(y_i - y_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2}\right]}$$

Cross section area

$$A_{eff} := \sum_{i=1}^{rows(y)-1} dA_i$$

$A = 1.06 \cdot 10^4 \cdot mm^2$

$A_{eff} = 1.002 \cdot 10^4 \cdot mm^2$

First moment of area

$$S_y := \sum_{i=1}^{rows(y)-1} \left(z_i + z_{i-1}\right) \cdot \frac{dA_i}{2}$$

Gravity centre

$z_{gc} := \frac{S_y}{A_{eff}}$ $z_{gc} = -41.546 \cdot mm$

Effective cross section Second iteration

Node 10 and 13 is moved to the neutral axis from the first iteration

$z_{10} := z_{gc} + 0.01 \cdot mm$ $z_{13} := z_{gc} + 0.01 \cdot mm$

$y_{10} := y_5 - \frac{z_{10}}{z_{11}} \cdot (y_5 - y_{11})$ $y_{13} := -y_{10}$

5.4.3 (1) $\psi_i := 1$ $j := 9..11$ $\psi_j := \frac{z_8 - z_{gc}}{z_{11} - z_{gc}}$ $\psi_{j+4} := \psi_j$ $z_{d_i} := |z_i - z_{gc}|$ $max_z := max(z_d)$

(5.7 or 5.8) $g_i := if\left(\psi_i > -1, 0.7 + 0.30 \cdot \psi_i, \frac{0.8}{1 - \psi_i}\right)$ Outstand Figure 5.2 $g_i := if(T_i > 0, 1, g_i)$

5.4.3 (1) c) $k := 1..2$ $\beta_k := \frac{y_2 - y_0 - 0.5 \cdot t_{13}}{t_2}$ $\beta_{k+5} := \beta_k$ $l := 3..5$ $\beta_l := \frac{y_5 - y_2 - t_{13}}{t_4}$

$$\beta_j := if\left[t_{11} > 0, g_j \cdot \frac{\sqrt{(y_{11} - y_8)^2 + (z_{11} - z_8)^2}}{t_{11}}, 1\right] \qquad \beta_{j+4} := \beta_j$$

5.4.4 (5) $\varepsilon_o := \sqrt{\frac{250 \cdot MPa}{f_o}}$ $\varepsilon_i := \varepsilon_o \cdot \sqrt{\frac{max_z}{|z_{gc}|}}$ $\varepsilon_i := if\left[(z_i \geq z_{gc}) \cdot (z_{i-1} \geq z_{gc}), \varepsilon_i, 99\right]$ $\varepsilon_o = 1.118$

5.4.5 (3) a) or c) non heat treated, welded

$$\rho_{c_i} := if\left[T_i > 0, if\left[\frac{\beta_i}{\varepsilon_i} > 4, 8 \cdot \frac{\varepsilon_i}{\beta_i} - 16 \cdot \left(\frac{\varepsilon_i}{\beta_i}\right)^2, 1.0\right], if\left[\frac{\beta_i}{\varepsilon_i} > 15, 25 \cdot \frac{\varepsilon_i}{\beta_i} - 150 \cdot \left(\frac{\varepsilon_i}{\beta_i}\right)^2, 1.0\right]\right]$$

Effective thickness $t_{eff} := \overrightarrow{(\rho_c \cdot t)}$

Heat Affected Zone

$t_{eff_n} := if(t_{eff_n} > t_{haz.f}, t_{haz.f}, t_{eff_n})$ $\quad t_{eff_{8-n}} := t_{eff_n}$

$t_{eff_9} := if(t_{eff_9} > t_{haz.w}, t_{haz.w}, t_{eff_9})$ $\quad t_{eff_{15}} := t_{eff_9}$

$i =$	$T_i =$	$\psi_i =$	$g_i =$	$\beta_i =$	$\varepsilon_i =$	$\frac{\beta_i}{\varepsilon_i} =$	$\rho_{c_i} =$	$\frac{t_{eff_i}}{mm} =$
1	1	1	1	3	1.315	2.282	1	15
2	1	1	1	3	1.315	2.282	1	12
3	0	1	1	11.6	1.315	8.823	1	12
4	0	1	1	11.6	1.315	8.823	1	15
5	0	1	1	11.6	1.315	8.823	1	12
6	1	1	1	3	1.315	2.282	1	12
7	1	1	1	3	1.315	2.282	1	15
8	0	1	1	0	1.315	0	1	0
9	0	-0.723	0.483	2.776	1.315	2.112	1	14.4
10	0	-0.723	0.483	2.776	1.315	2.112	1	18
11	0	-0.723	0.483	2.776	99	0.028	1	18
12	0	1	1	0	99	0	1	18
13	0	-0.723	0.483	2.776	99	0.028	1	18
14	0	-0.723	0.483	2.776	1.315	2.112	1	18
15	0	-0.723	0.483	2.776	1.315	2.112	1	14.4

Max slenderness - Internal elements

$\beta_{e_i} := if\left(T_i > 0, 0, \frac{\beta_i}{\varepsilon_i}\right)$

$\beta_{Imax} := max(\beta_e)$

$\beta_{Imax} = 8.823$

Max slenderness - Outstands

$\beta_{e_i} := if\left(T_i > 0, \frac{\beta_i}{\varepsilon_i}, 0\right)$

$\beta_{Omax} := max(\beta_e)$

$\beta_{Omax} = 2.282$

Cross section class (5.15)

$class_I := if(\beta_{Imax} \le 7, 1, if(\beta_{Imax} \le 11, 2, if(\beta_{Imax} \le 15, 3, 4)))$ $\quad class_I = 2$

$class_O := if(\beta_{Omax} \le 2, 1, if(\beta_{Omax} \le 3, 2, if(\beta_{Omax} \le 4, 3, 4)))$ $\quad class_O = 2$

$class := if(class_I > class_O, class_I, class_O)$ $\quad class = 2$

Area of effective thickness elements

$$dA_i := \overrightarrow{\left[t_{eff_i} \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}\right]}$$

Area of effective cross section

$$A_{eff} := \sum_{i=1}^{rows(y)-1} dA_i$$

$A = 1.06 \cdot 10^4 \cdot mm^2$

$A_{eff} = 1.002 \cdot 10^4 \cdot mm^2$

First moment of area. Gravity centre

$$S_y := \sum_{i=1}^{rows(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$z_{gc} := \frac{S_y}{A_{eff}}$

$z_{gc} = -41.546 \cdot mm$

Second moment of area of effective cross section

$$I_y := \sum_{i=1}^{rows(y)-1} \left[(z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$$

$$I_{eff} := I_y - A_{eff} \cdot z_{gc}^2$$

$$I_{eff} = 1.809 \cdot 10^7 \circ mm^4$$

Section modulus

$$z_{d_i} := \left| z_i - z_{gc} \right| \qquad max_{z.eff} := max(z_d) \qquad W_{eff} := \frac{I_{eff}}{max_{z.eff}} \qquad W_{eff} = 3.149 \cdot 10^5 \circ mm^3$$

Compare gross section

$$z_{d_i} := \left| z_i - z_{gc.gr} \right| \qquad max_{z.gr} := max(z_d) \qquad W_{gr} := \frac{I_{y.gr}}{max_{z.gr}} \qquad W_{gr} = 3.172 \cdot 10^5 \circ mm^3$$

Heat Affected Zone

Reduced thickness in HAZ $h := 2..3 \qquad t_h := \rho_{haz} \cdot t_h \qquad t_{h+3} := t_h \qquad t_9 := \rho_{haz} \cdot t_h \qquad t_{15} := \rho_{haz} \cdot t_h$

Area

$$dA_i := \overrightarrow{\left[t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]}$$

Area of effective cross section

$$A_{ele} := \sum_{i=1}^{rows(y)-1} dA_i$$

$$A = 1.06 \cdot 10^4 \circ mm^2$$

$$A_{ele} = 9.736 \cdot 10^3 \circ mm^2$$

First moment of area. Gravity centre

$$S_y := \sum_{i=1}^{rows(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$$z_{gc.e} := \frac{S_y}{A_{ele}}$$

$$z_{gc.e} = -42.351 \circ mm$$

Second moment of area of effective cross section

$$I_y := \sum_{i=1}^{rows(y)-1} \left[(z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$$

$$I_{ele} := I_y - A_{ele} \cdot z_{gc.e}^2$$

$$I_{ele} = 1.785 \cdot 10^7 \circ mm^4$$

Section modulus

$$z_{d_i} := \left| z_i - z_{gc.e} \right| \qquad max_{z.ele} := max(z_d) \qquad W_{ele} := \frac{I_{ele}}{max_{z.ele}} \qquad W_{ele} = 3.151 \cdot 10^5 \circ mm^3$$

$$W_{el} = 3.172 \cdot 10^5 \circ mm^3$$

Plastic section modulus

Area of effective cross section

$$A_{pl} := \left[\sum_{i=1}^{rows(y)-1} \left[t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right] \right]$$

$$A = 1.06 \cdot 10^4 \circ mm^2$$

$$A_{pl} = 9.736 \cdot 10^3 \circ mm^2$$

Cross section area of upper part

$$A_{up} := \left[\sum_{i=1}^{7} \left[t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right] \right] + 2 \cdot \left[t_9 \cdot \sqrt{(y_9 - y_8)^2 + (z_9 - z_8)^2} \right]$$

$$A_{up} = 4.716 \cdot 10^3 \circ mm^2$$

Move node 8 to plastic neutral axis

$$z_{10} := z_9 - \frac{\frac{A}{2} - A_{up}}{t_{11} + t_{13}} \qquad z_9 = -28.711 \circ mm \qquad z_{13} := z_{10} \qquad z_{10} = -44.934 \circ mm$$

$$y_{10} := y_5 - \frac{z_{10}}{z_{11}} \cdot (y_5 - y_{11}) \qquad y_{13} := -y_{10} \qquad y_{10} = 82.384 \circ mm$$

Plastic section modulus

$$W_{ple} := \sum_{i=1}^{rows(y)-1} \frac{\left|z_i + z_{i-1}\right|}{2} \cdot \left[t_i \cdot \sqrt{\left(y_i - y_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2}\right]$$

$W_{ple} = 4.123 \cdot 10^5 \cdot mm^3$

$\frac{W_{ple}}{W_{el}} = 1.3$

Shape factor

(5.15)

$$\alpha_{3I} := \frac{W_{ele}}{W_{el}} \cdot \left[1 + \frac{15 - \beta_{Imax}}{15 - 11} \cdot \left(\frac{W_{ple}}{W_{ele}} - 1\right)\right]$$

$$\alpha_{3O} := \frac{W_{ele}}{W_{el}} \cdot \left[1 + \frac{4 - \beta_{Omax}}{4 - 3} \cdot \left(\frac{W_{ple}}{W_{ele}} - 1\right)\right]$$

$\alpha_{3I} = 1.467$

$\alpha_{3O} = 1.52$

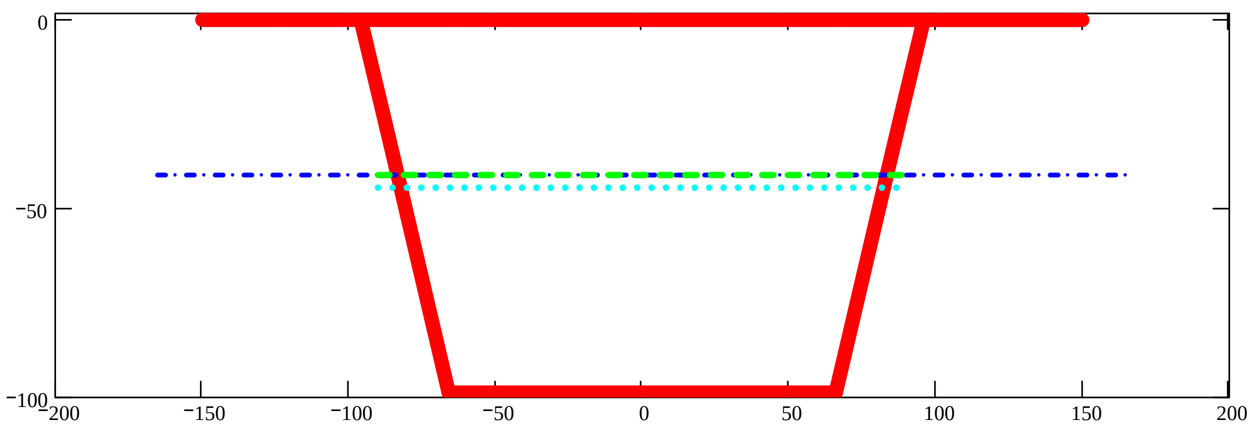
$\alpha_3 := if\left(\alpha_{3I} < \alpha_{3O}, \alpha_{3I}, \alpha_{3O}\right)$

$\alpha_3 = 1.467$

$$\alpha := if\left(class < 3, \frac{W_{ple}}{W_{el}}, if\left(class < 4, \alpha_3, \frac{W_{eff}}{W_{el}}\right)\right)$$

$class = 2$

$\alpha = 1.3$



Elastic neutral axis:
- first iteration dashed-dotted line
- second iteration dashed line

Plastic neutral axis dotted line

Bending moment resistance

(5.14)

$$M_{y.Rd} := \alpha \cdot W_{el} \cdot \frac{f_o}{\gamma_{M1}}$$

$$M_{y.Rd} = 75 \circ kN \cdot m$$

$$W_{el} = 3.172 \cdot 10^5 \circ mm^3$$

$$W_{ple} = 4.123 \cdot 10^5 \circ mm^3$$

Cross section class

$class = 2$

$\alpha = 1.3$

$$A_{eff} = 1.002 \cdot 10^4 \circ mm^2$$

$$I_{eff} = 1.809 \cdot 10^7 \circ mm^4$$