

**TALAT Lecture 2301**

**Design of Members**

**Bending Moment**

**Example 4.4 : Bending moment resistance of welded hollow section with outstands. Class 4 cross section**

10 pages

Advanced Level

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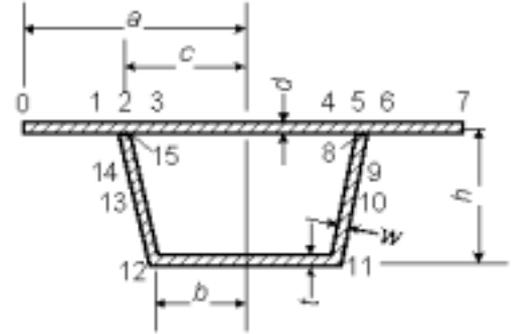
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## Example 4.4. Bending moment resistance of welded hollow section with outstands. Class 4 cross section

Half top flange	$a := 200\text{-mm}$	Half hollow width	$c := 96\text{-mm}$	$f_o := 200\text{-MPa}$
Half bottom flange	$b := 96\text{-mm}$	Width of HAZ	$haz := 30\text{-mm}$	$\gamma_{M\bar{F}} = 1.1$
Section dept	$h := 99\text{-mm}$	HAZ reduction	$\rho_{haz} = 0.8$	$MPa = 10^6 \cdot Pa$
Flange thickness	$d := 6\text{-mm}$			
Web thickness	$w := 8\text{-mm}$			
Bottom flange t.	$t := 8\text{-mm}$			
Type of element				
- Internal element:	$T = 0$			
- Outstand:	$T > 0$			
	$o := 0\text{-mm}$			

$$bz := c - \frac{haz}{\sqrt{(c-b)^2 + h^2}} \cdot (c-b)$$

$$hz := \frac{haz}{\sqrt{(c-b)^2 + h^2}} \cdot h$$



Nodes, co-ordinates (y, z), thickness (t) and types (T)

$i :=$	$y :=$	$z :=$	$t :=$	$T :=$
0	-a	o	o	0
1	-c - haz	o	d	1
2	-c	o	d	1
3	-c + haz	o	d	0
4	c - haz	o	d	0
5	c	o	d	0
6	c + haz	o	d	1
7	a	o	d	1
8	c	o	o	0
9	bz	-hz	w	0
10	0.5 · (c + b)	-0.5 · h	w	0
11	b	-h	w	0
12	-b	-h	t	0
13	-0.5 · (c + b)	-0.5 · h	w	0
14	-bz	-hz	w	0
15	-c	o	w	0

Comments:

Node 10 and 13 indicate shift in neutral axis. Element 1-2, 2-3, 4-5, 5-6, 8-9 and 14-15 define HAZs.

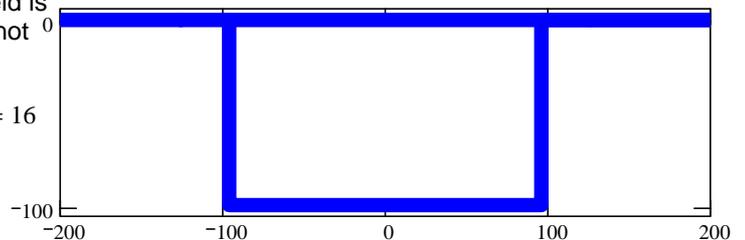
This solution of the example is comprehensive because:

1. Calculations are shown in every detail
2. It covers all classes of cross section
3. The possibility to increase the effective thickness for elements not fully stressed is shown. The iteration procedure is then mandatory. However, in this example, the result is almost exactly the same as if a simplified method was used, that's because there is no reduction of the web thickness due to local buckling.

The beam is composed by two extrusions. The weld is close to the neutral axis why HAZ softening does not influence the moment resistance.

Nodes  $i := 1..rows(y) - 1$   $rows(y) = 16$

Area of cross section elements  $dA_i := t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2}$



Cross section area

$$A := \sum_{i=1}^{\text{rows}(y) - 1} dA_i$$

$$A = 5.52 \cdot 10^3 \cdot \text{mm}^2$$

First moment of area  
Gravity centre

$$S_y := \sum_{i=1}^{\text{rows}(y) - 1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$$z_{gc} := \frac{S_y}{A}$$

$$z_{gc} = -41.752 \cdot \text{mm}$$

Second moment of area

$$I_y := \sum_{i=1}^{\text{rows}(y) - 1} \left[ (z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$$

$$z_{gc,gr} := z_{gc}$$

$$I_y := I_y - A \cdot z_{gc}^2$$

$$I_{y,gr} := I_y$$

$$I_y = 1.061 \cdot 10^7 \cdot \text{mm}^4$$

Elastic section modulus

$$W_{el} := \text{if} \left( \max(z - z_{gc}) > \left| \min(z - z_{gc}) \right|, \frac{I_y}{\max(z - z_{gc})}, \frac{I_y}{\left| \min(z - z_{gc}) \right|} \right)$$

$$W_{el} = 1.853 \cdot 10^5 \cdot \text{mm}^3$$

Torsion constant for open parts

$$I_{tu} := \sum_{i=1}^{\text{rows}(y) - 1} dA_i \cdot \frac{(t_i)^2}{3}$$

$$I_{tu} = 9.536 \cdot 10^4 \cdot \text{mm}^4$$

Hollow part, (nodes 2, 5, 11, 12 and 15)

$$y_h := \begin{bmatrix} y_2 \\ y_5 \\ y_{11} \\ y_{12} \\ y_{15} \end{bmatrix} \quad y_h = \begin{bmatrix} -96 \\ 96 \\ 96 \\ -96 \\ -96 \end{bmatrix} \cdot \text{mm} \quad z_h := \begin{bmatrix} z_2 \\ z_5 \\ z_{11} \\ z_{12} \\ z_{15} \end{bmatrix} \quad z_h = \begin{bmatrix} 0 \\ 0 \\ -99 \\ -99 \\ 0 \end{bmatrix} \cdot \text{mm} \quad t_h := \begin{bmatrix} t_1 \\ t_4 \\ t_{11} \\ t_{12} \\ t_{14} \end{bmatrix} \quad t_h = \begin{bmatrix} 6 \\ 6 \\ 8 \\ 8 \\ 8 \end{bmatrix} \cdot \text{mm}$$

Area within mid-line

$$A_{tu} := \sum_{i=1}^{\text{rows}(y_h) - 1} 0.5 \cdot (y_{h_i} - y_{h_{i-1}}) \cdot (z_{h_i} + z_{h_{i-1}})$$

$$A_{tu} = 1.901 \cdot 10^4 \cdot \text{mm}^2$$

Sum of l/t

$$Dn := \sum_{i=1}^{\text{rows}(y_h) - 1} \text{if} \left[ t_i > 0, \frac{\sqrt{(y_{h_i} - y_{h_{i-1}})^2 + (z_{h_i} - z_{h_{i-1}})^2}}{t_{h_i}}, 10^{22} \right]$$

$$Dn = 80.75$$

Torsion constant for closed part

$$I_t := \frac{4 \cdot A_{tu}^2}{Dn}$$

$$I_t = 1.79 \cdot 10^7 \cdot \text{mm}^4$$

Torsion constant whole section

$$I_t := I_t + I_{tu}$$

$$I_t = 1.799 \cdot 10^7 \cdot \text{mm}^4$$

Torsion resistance

$$W_v := 2 \cdot A_{tu} \cdot \min(t_h)$$

$$\min(t_h) = 6 \cdot \text{mm}$$

$$W_v = 2.281 \cdot 10^5 \cdot \text{mm}^3$$

$$z_{gc} = -41.752 \cdot \text{mm}$$

## Effective cross section First iteration

5.4.3 (1)  $\psi_i := 1 \quad j := 9..11 \quad \psi_j := \frac{z_8 - z_{gc}}{z_{11} - z_{gc}} \quad \psi_{j+4} := \psi_j \quad z_{d_i} := |z_i - z_{gc}|$   
 $\max_z := \max(z_d)$

(5.7 or 5.8)  $g_i := \text{if} \left( \psi_i > -1, 0.7 + 0.30 \cdot \psi_i, \frac{0.8}{1 - \psi_i} \right)$  **Outstand Figure 5.2**  $g_i := \text{if}(T_i > 0, 1, g_i)$

5.4.3 (1) c)  $k := 1..2 \quad \beta_k := \frac{y_2 - y_0 - 0.5 \cdot t_{13}}{t_2} \quad \beta_{k+5} := \beta_k \quad y_2 = -96 \text{ mm}$   
 $l := 3..5 \quad \beta_l := \frac{y_5 - y_2 - t_{13}}{t_4} \quad y_0 = -200 \text{ mm}$   
 $\frac{y_2 - y_0}{t_2} = 17.333$

$\beta_j := \text{if} \left[ t_{11} > 0, g_j \cdot \frac{\sqrt{(y_{11} - y_8)^2 + (z_{11} - z_8)^2}}{t_{11}}, 1 \right] \quad \beta_{j+4} := \beta_j$

5.4.4 (5)  $\varepsilon_o := \sqrt{\frac{250 \text{ MPa}}{f_o}} \quad \varepsilon_i := \varepsilon_o \cdot \sqrt{\frac{\max_z}{z_{gc}}} \quad \varepsilon_i := \text{if} \left[ (z_i \geq z_{gc}) \cdot (z_{i-1} \geq z_{gc}), \varepsilon_i, 99 \right] \quad \varepsilon_o = 1.118$

5.4.5 (3) a) or c) non heat treated, welded  
 $\rho_{\varepsilon_i} := \text{if} \left[ T_i > 0, \text{if} \left[ \frac{\beta_i}{\varepsilon_i} > 4, 8 \cdot \frac{\varepsilon_i}{\beta_i} - 16 \cdot \left( \frac{\varepsilon_i}{\beta_i} \right)^2, 1.0 \right], \text{if} \left[ \frac{\beta_i}{\varepsilon_i} > 15, 25 \cdot \frac{\varepsilon_i}{\beta_i} - 150 \cdot \left( \frac{\varepsilon_i}{\beta_i} \right)^2, 1.0 \right] \right]$

Effective thickness  $t_{eff} := \left( \rho \cdot t \right)$

## Heat Affected Zone

Flange  $t_{haz.f} := \rho \cdot haz_t2 \quad t_{haz.f} = 4.8 \text{ mm}$

Web  $t_{haz.w} := \rho \cdot haz_t9 \quad t_{haz.w} = 6.4 \text{ mm}$

$n := 2..3 \quad t_{eff_n} := \text{if} \left( t_{eff_n} > t_{haz.f}, t_{haz.f}, t_{eff_n} \right) \quad t_{eff_{8-n}} := t_{eff_n}$   
 $t_{eff_9} := \text{if} \left( t_{eff_9} > t_{haz.w}, t_{haz.w}, t_{eff_9} \right) \quad t_{eff_{15}} := t_{eff_9}$

Comment:

$\varepsilon_i$  was given a large value (99)  
 for elements entirely on  
 the tension side

$\psi_i =$	$g_i =$	$\beta_i =$	$\varepsilon_i =$	$\frac{\beta_i}{\varepsilon_i} =$	$\rho_{\varepsilon_i} =$	$\frac{t_{eff_i}}{\text{mm}} =$
1	1	16.667	1.309	12.731	0.53	3.178
1	1	16.667	1.309	12.731	0.53	3.178
1	1	30.667	1.309	23.425	0.794	4.763
1	1	30.667	1.309	23.425	0.794	4.763
1	1	16.667	1.309	12.731	0.53	3.178
1	1	16.667	1.309	12.731	0.53	3.178
1	1	0	1.309	0	1	0
-0.729	0.481	5.955	1.309	4.549	1	6.4
-0.729	0.481	5.955	99	0.06	1	8
-0.729	0.481	5.955	99	0.06	1	8
1	1	0	99	0	1	8
-0.729	0.481	5.955	99	0.06	1	8
-0.729	0.481	5.955	99	0.06	1	8
-0.729	0.481	5.955	1.309	4.549	1	6.4

Area of effective thickness elements

$$dA_i := \left[ t_{eff_i} \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]$$

Cross section area

$$A_{eff} := \sum_{i=1}^{rows(y)-1} dA_i$$

$$A = 5.52 \cdot 10^3 \cdot mm^2$$

First moment of area

$$S_y := \sum_{i=1}^{rows(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$$A_{eff} = 4.6 \cdot 10^3 \cdot mm^2$$

$$z_{gc} := \frac{S_y}{A_{eff}} \quad z_{gc} = -49.794 \cdot mm$$

Gravity centre

## Effective cross section Second iteration

Node 10 and 13 is moved to the neutral axis from the first iteration

$$z_{10} := z_{gc} + 0.01 \cdot mm$$

$$z_{13} := z_{gc} + 0.01 \cdot mm$$

$$y_{10} := y_5 - \frac{z_{10}}{z_{11}} \cdot (y_5 - y_{11})$$

$$y_{13} := -y_{10}$$

5.4.3 (1)

$$\psi_i := 1 \quad j := 9..11$$

$$\psi_j := \frac{z_8 - z_{gc}}{z_{11} - z_{gc}} \quad \psi_{j+4} := \psi_j$$

$$z_{d_i} := |z_i - z_{gc}|$$

$$max_z := max(z_d)$$

(5.7 or 5.8)

$$g_i := if\left(\psi_i > -1, 0.7 + 0.30 \cdot \psi_i, \frac{0.8}{1 - \psi_i}\right)$$

Outstand Figure 5.2

$$g_i := if(T_i > 0, 1, g_i)$$

5.4.3 (1) c)

$$k := 1..2 \quad \beta_k := \frac{y_2 - y_0 - 0.5 \cdot t_{13}}{t_2} \quad \beta_{k+5} := \beta_k \quad l := 3..5 \quad \beta_l := \frac{y_5 - y_2 - t_{13}}{t_4}$$

$$\beta_j := if\left[t_{11} > 0, g_j \cdot \frac{\sqrt{(y_{11} - y_8)^2 + (z_{11} - z_8)^2}}{t_{11}}, 1\right] \quad \beta_{j+4} := \beta_j$$

5.4.4 (5)

$$\varepsilon_o := \sqrt{\frac{250 \cdot MPa}{f_o}} \quad \varepsilon_i := \varepsilon_o \cdot \sqrt{\frac{max_z}{|z_{gc}|}} \quad \varepsilon_i := if\left[(z_i \geq z_{gc}) \cdot (z_{i-1} \geq z_{gc}), \varepsilon_i, 99\right] \quad \varepsilon_{\bar{o}} = 1.118$$

5.4.5 (3)

a) or c)  
non heat treated, welded

$$\rho_{\bar{t}} := if\left[T_i > 0, if\left[\frac{\beta_i}{\varepsilon_i} > 4, 8 \cdot \frac{\varepsilon_i}{\beta_i} - 16 \cdot \left(\frac{\varepsilon_i}{\beta_i}\right)^2, 1.0\right], if\left[\frac{\beta_i}{\varepsilon_i} > 15, 25 \cdot \frac{\varepsilon_i}{\beta_i} - 150 \cdot \left(\frac{\varepsilon_i}{\beta_i}\right)^2, 1.0\right]\right]$$

Effective thickness

$$t_{eff} := \overrightarrow{(\rho \cdot \bar{t})}$$

## Heat Affected Zone

$$t_{eff_n} := \text{if}(t_{eff_n} > t_{haz.f}, t_{haz.f}, t_{eff_n}) \quad t_{eff_{8-n}} := t_{eff_n}$$

$$t_{eff_9} := \text{if}(t_{eff_9} > t_{haz.w}, t_{haz.w}, t_{eff_9}) \quad t_{eff_{15}} := t_{eff_9}$$

$i =$	$T_i =$	$\psi_i =$	$g_i =$	$\beta_i =$	$\varepsilon_i =$	$\frac{\beta_i}{\varepsilon_i} =$	$\rho_{\varepsilon} =$	$\frac{t_{eff_i}}{mm} =$
1	1	1	1	16.667	1.118	14.907	0.465	2.788
2	1	1	1	16.667	1.118	14.907	0.465	2.788
3	0	1	1	30.667	1.118	27.429	0.712	4.272
4	0	1	1	30.667	1.118	27.429	0.712	4.272
5	0	1	1	30.667	1.118	27.429	0.712	4.272
6	1	1	1	16.667	1.118	14.907	0.465	2.788
7	1	1	1	16.667	1.118	14.907	0.465	2.788
8	0	1	1	0	1.118	0	1	0
9	0	-1.012	0.398	4.921	1.118	4.401	1	6.4
10	0	-1.012	0.398	4.921	1.118	4.401	1	8
11	0	-1.012	0.398	4.921	99	0.05	1	8
12	0	1	1	0	99	0	1	8
13	0	-1.012	0.398	4.921	99	0.05	1	8
14	0	-1.012	0.398	4.921	1.118	4.401	1	8
15	0	-1.012	0.398	4.921	1.118	4.401	1	6.4

Max slenderness  
- Internal elements

$$\beta_{i_e} := \text{if}(T_i > 0, 0, \frac{\beta_i}{\varepsilon_i})$$

$$\beta_{Imax} := \max(\beta_{e_i})$$

$$\beta_{Imax} = 27.429$$

Max slenderness  
- Outstands

$$\beta_{i_e} := \text{if}(T_i > 0, \frac{\beta_i}{\varepsilon_i}, 0)$$

$$\beta_{Omax} := \max(\beta_{e_i})$$

$$\beta_{Omax} = 14.907$$

Cross section class (5.15)

$$class_I := \text{if}(\beta_{Imax} \leq 7, 1, \text{if}(\beta_{Imax} \leq 11, 2, \text{if}(\beta_{Imax} \leq 15, 3, 4))) \quad class_I = 4$$

$$class_O := \text{if}(\beta_{Omax} \leq 2, 1, \text{if}(\beta_{Omax} \leq 3, 2, \text{if}(\beta_{Omax} \leq 4, 3, 4))) \quad class_O = 4$$

$$class := \text{if}(class_I > class_O, class_I, class_O) \quad class = 4$$

Area of effective thickness elements

$$dA_i := \left[ t_{eff_i} \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]$$

Area of effective cross section

$$A_{eff} := \sum_{i=1}^{rows(y)-1} dA_i$$

$$A = 5.52 \cdot 10^3 \cdot mm^2$$

$$A_{eff} = 4.424 \cdot 10^3 \cdot mm^2$$

First moment of area.  
Gravity centre

$$S_y := \sum_{i=1}^{rows(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$$z_{gc} := \frac{S_y}{A_{eff}}$$

$$z_{gc} = -51.768 \cdot mm$$

Second moment of area of effective cross section

$$I_y := \sum_{i=1}^{rows(y)-1} \left[ (z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$$

$$I_{eff} := I_y - A_{eff} \cdot z_{gc}^2$$

$$I_{eff} = 8.344 \cdot 10^6 \cdot mm^4$$

Section modulus

$$z_{d_i} := |z_i - z_{gc}| \quad \max_{z,eff} := \max(z_d) \quad W_{eff} := \frac{I_{eff}}{\max_{z,eff}}$$

$$W_{eff} = 1.612 \cdot 10^5 \cdot mm^3$$

Compare gross section

$$z_{d_i} := |z_i - z_{gc,gr}| \quad \max_{z,gr} := \max(z_d) \quad W_{gr} := \frac{I_{y,gr}}{\max_{z,gr}}$$

$$W_{gr} = 1.853 \cdot 10^5 \cdot mm^3$$

## Heat Affected Zone

Reduced thickness in HAZ

$$h := 2..3 \quad t_h := \rho \cdot ha \cdot z_h \quad t_{h+3} := t_h \quad t_9 := \rho \cdot ha \cdot z_h \quad t_{15} := \rho \cdot ha \cdot z_h$$

Area

$$dA_i := \left[ t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]$$

Area of effective cross section

$$A_{ele} := \sum_{i=1}^{rows(y)-1} dA_i$$

$$A = 5.52 \cdot 10^3 \cdot mm^2$$

$$A_{ele} = 5.126 \cdot 10^3 \cdot mm^2$$

First moment of area. Gravity centre

$$S_y := \sum_{i=1}^{rows(y)-1} (z_i + z_{i-1}) \cdot \frac{dA_i}{2}$$

$$z_{gc,e} := \frac{S_y}{A_{ele}}$$

$$z_{gc,e} = -44.228 \cdot mm$$

Second moment of area of effective cross section

$$I_y := \sum_{i=1}^{rows(y)-1} \left[ (z_i)^2 + (z_{i-1})^2 + z_i \cdot z_{i-1} \right] \cdot \frac{dA_i}{3}$$

$$I_{ele} := I_y - A_{ele} \cdot z_{gc,e}^2$$

$$I_{ele} = 1.013 \cdot 10^7 \cdot mm^4$$

Section modulus

$$z_{d_i} := |z_i - z_{gc,e}| \quad \max_{z,ele} := \max(z_d) \quad W_{ele} := \frac{I_{ele}}{\max_{z,ele}}$$

$$W_{ele} = 1.849 \cdot 10^5 \cdot mm^3$$

$$W_{el} = 1.853 \cdot 10^5 \cdot mm^3$$

## Plastic section modulus

Area of effective cross section	$A_{pl} := \left[ \sum_{i=1}^{rows(y)-1} \left[ t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right] \right]$	$A = 5.52 \cdot 10^3 \cdot mm^2$ $A_{pl} = 5.126 \cdot 10^3 \cdot mm^2$
Cross section area of upper part	$A_{up} := \left[ \sum_{i=1}^7 \left[ t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right] \right] + 2 \cdot \left[ t_9 \cdot \sqrt{(y_9 - y_8)^2 + (z_9 - z_8)^2} \right]$	$A_{up} = 2.486 \cdot 10^3 \cdot mm^2$
Move node 8 to plastic neutral axis	$z_{10} := z_9 - \frac{\frac{A}{2} - A_{up}}{t_{11} + t_{13}}$ $y_{10} := y_5 - \frac{z_{10}}{z_{11}} \cdot (y_5 - y_{11})$	$z_9 = -30 \cdot mm$ $z_{13} := z_{10}$ $y_{13} := -y_{10}$
Plastic section modulus	$W_{ple} := \sum_{i=1}^{rows(y)-1} \frac{ z_i + z_{i-1} }{2} \cdot \left[ t_i \cdot \sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]$	$W_{ple} = 2.267 \cdot 10^5 \cdot mm^3$ $\frac{W_{ple}}{W_{el}} = 1.224$
Shape factor		

# Shape factor

" el

$$(5.15) \quad \alpha = \frac{W_{ele}}{W_{el}} \left[ 1 + \frac{15 - \beta}{15 - 11} \cdot \frac{I_{max}}{W_{ele}} \cdot \left( \frac{W_{ple}}{W_{ele}} - 1 \right) \right]$$

$$\alpha = \frac{W_{ele}}{W_{el}} \left[ 1 + \frac{4 - \beta}{4 - 3} \cdot \frac{O_{max}}{W_{ele}} \cdot \left( \frac{W_{ple}}{W_{ele}} - 1 \right) \right]$$

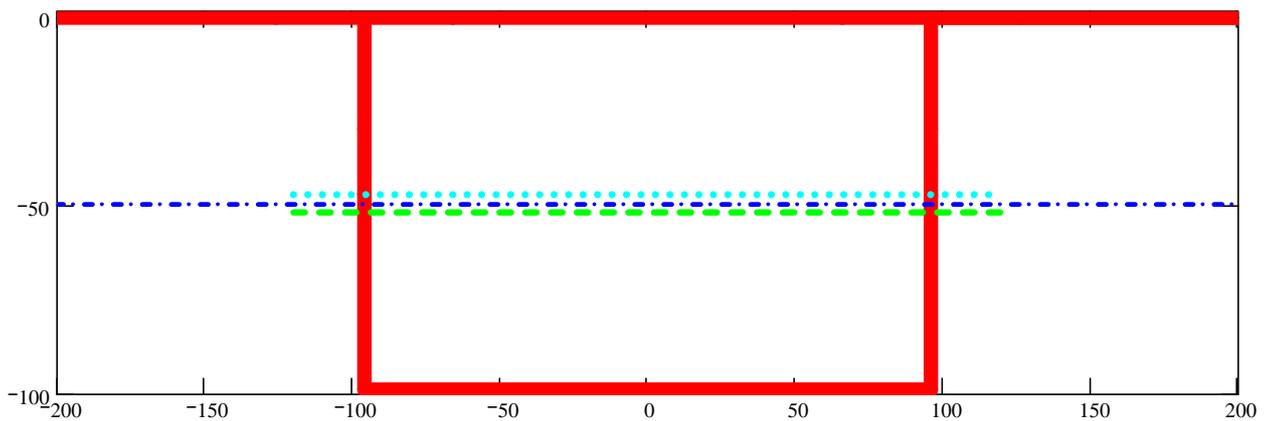
$$\alpha = 0.296$$

$$\alpha = 1.465$$

$$\alpha = \begin{cases} \alpha_{3I} & \text{if } (\alpha_{3I} < \alpha_{3O}, \alpha_{3I} < \alpha_{3O}) \\ \alpha_{3O} & \text{if } (\alpha_{3O} < \alpha_{3I}, \alpha_{3O} < \alpha_{3I}) \end{cases}$$

$$\alpha = \begin{cases} \frac{W_{ple}}{W_{el}} & \text{if } (class < 3) \\ \frac{W_{eff}}{W_{el}} & \text{if } (class < 4) \end{cases}$$

$$class = 4 \quad \alpha = 0.87$$



Elastic neutral axis:  
 - first iteration   dashed-dotted line  
 - second iteration   dashed line  
 Plastic neutral axis   dotted line

## Bending moment resistance

$$(5.14) \quad M_{y,Rd} := \alpha \cdot W_{el} \frac{f_o}{\gamma_{M1}}$$

$$M_{y,Rd} = 29.3 \cdot kN \cdot m$$

$$W_{el} = 1.853 \cdot 10^5 \cdot mm^3$$

$$W_{ple} = 2.267 \cdot 10^5 \cdot mm^3$$

Cross section  
class  $class = 4$

$$\alpha = 0.87$$

$$A_{eff} = 4.424 \cdot 10^3 \cdot mm^2$$

$$I_{eff} = 8.344 \cdot 10^6 \cdot mm^4$$

