TALAT Lecture 2301

Design of Members

Axial Force

Example 5.3 : Resistance of cross section with radiating outstands

6 pages

Advanced Level

prepared by Torsten Höglund, Royal Institute of Technology, Stockholm

Example 5.3. Resistance of cross section with radiating outstands

Flange width	$b := 80 \cdot mm$	Length	$L := 1200 \cdot mm$
Inner circle	$c := 18 \cdot mm$	$f_o := 300 \cdot MPa$ $\gamma_{M1} := 1.0$	$kN \equiv 1000 \cdot newton$
Inner thickness	$ti := 10 \cdot mm$	$E := 70000 \cdot MPa$ $G := 27000 \cdot MPa$	$MPa \equiv 10^6 \cdot Pa$
Outer thickness	$to := 5 \cdot mm$		

$$k := 1,4..16 \quad \phi_{k-1} := \frac{\pi}{4} \cdot \frac{k-1}{3} - \frac{\pi}{4} \quad \phi_k := \phi_{k-1}$$

$$\phi_{k+1} := \frac{\pi}{4} \cdot \frac{k+2}{3} - \frac{\pi}{4} \quad \phi_{18} := \phi_{17}$$

Nodes no., co-ordinates, thickness

$$i := 0..16 \quad y_{k-1} := b \cdot cos(\phi_{k-1}) \quad z_{k-1} := b \cdot sin(\phi_{k-1})$$
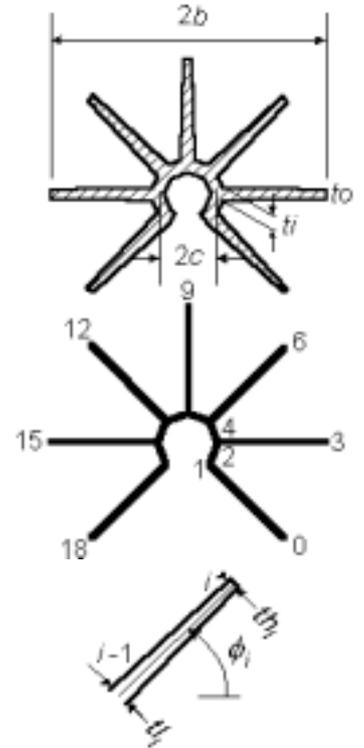$$y_k := c \cdot cos(\phi_k) \quad z_k := c \cdot sin(\phi_k)$$
$$y_{k+1} := c \cdot cos(\phi_{k+1}) \quad z_{k+1} := c \cdot sin(\phi_{k+1})$$

$$y_{18} := y_{12} \quad z_{18} := z_0$$

$$tl_{k-1} := o \quad th_{k-1} := o \quad tl_k := to \quad th_k := ti$$

$$tl_{k+1} := ti \cdot 0.8 \quad th_{k+1} := tl_{k+1} \quad tl_{18} := ti \quad th_{18} := to$$

$i =$	$\phi_i =$	$\dfrac{y_i}{mm} =$	$\dfrac{z_i}{mm} =$	$\dfrac{tl_i}{mm}$	$\dfrac{th_i}{mm} =$
1	-0.785	12.728	-12.728	5	10
2	0	18	0	8	8
3	0	80	0	0	0
4	0	18	0	5	10
5	0.785	12.728	12.728	8	8
6	0.785	56.569	56.569	0	0
7	0.785	12.728	12.728	5	10
8	1.571	0	18	8	8
9	1.571	0	80	0	0
10	1.571	0	18	5	10
11	2.356	-12.728	12.728	8	8
12	2.356	-56.569	56.569	0	0
13	2.356	-12.728	12.728	5	10
14	3.142	-18	0	8	8
15	3.142	-80	0	0	0
16	3.142	-18	0	5	10
17	3.927	-12.728	-12.728	8	8
18	3.927	-56.569	-56.569	10	5
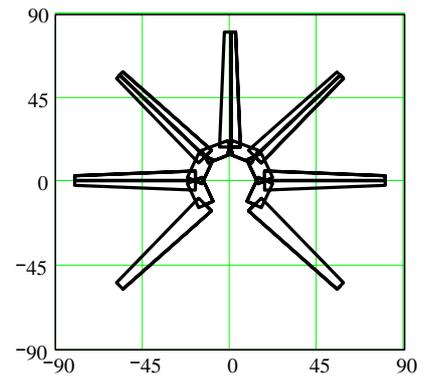
Nodes $\quad i := 1..rows(y) - 1$

Cross-section constants, varying thickness

Sectorial co-ordinates

$$\omega_0 := 0 \cdot mm^2$$

$$\omega_{0_i} = y_{i-1} \cdot z_i - y_i \cdot z_{i-1}$$

$$\omega_i := \omega_{i-1} + \omega_{0_i}$$

$$Dt_i := th_i - tl_i \qquad t1_i := tl_i + \frac{Dt_i}{2} \qquad t2_i := \frac{tl_i}{2} + \frac{Dt_i}{3} \qquad t3_i := \frac{tl_i}{3} + \frac{Dt_i}{4}$$

Length of elements

$$l_i := \overrightarrow{\left[\sqrt{\left(y_i - y_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2}\right]}$$

Area

$$A := \sum_{i=1}^{rows(y)-1} \frac{tl_i + th_i}{2} \cdot l_i \qquad\qquad A = 3.916 \cdot 10^3 \circ mm^2$$

First moments of area

$$S_z := \sum_{i=1}^{rows(y)-1} \left[y_{i-1} \cdot t1_i + \left(y_i - y_{i-1} \right) \cdot t2_i \right] \cdot l_i \qquad\qquad S_z = 1.186 \cdot 10^{-11} \circ mm^3$$

$$S_y := \sum_{i=1}^{rows(y)-1} \left[z_{i-1} \cdot t1_i + \left(z_i - z_{i-1} \right) \cdot t2_i \right] \cdot l_i \qquad\qquad S_y = 2.457 \cdot 10^4 \circ mm^3$$

$$S_\omega := \sum_{i=1}^{rows(y)-1} \left[\omega_{i-1} \cdot t1_i + \left(\omega_i - \omega_{i-1} \right) \cdot t2_i \right] \cdot l_i \qquad\qquad S_\omega = 2.692 \cdot 10^6 \circ mm^4$$

Second moments of area

$$I_z := \sum_{i=1}^{rows(y)-1} \left[\left(y_{i-1} \right)^2 \cdot t1_i + 2 \cdot y_{i-1} \cdot \left(y_i - y_{i-1} \right) \cdot t2_i + \left(y_i - y_{i-1} \right)^2 \cdot t3_i \right] \cdot l_i$$

$$I_z = 4.551 \cdot 10^6 \circ mm^4$$

$$I_y := \sum_{i=1}^{rows(y)-1} \left[\left(z_{i-1} \right)^2 \cdot t1_i + 2 \cdot z_{i-1} \cdot \left(z_i - z_{i-1} \right) \cdot t2_i + \left(z_i - z_{i-1} \right)^2 \cdot t3_i \right] \cdot l_i$$

$$I_y = 3.402 \cdot 10^6 \circ mm^4$$

$$I_\omega := \sum_{i=1}^{rows(y)-1} \left[\left(\omega_{i-1} \right)^2 \cdot t1_i + 2 \cdot \omega_{i-1} \cdot \left(\omega_i - \omega_{i-1} \right) \cdot t2_i + \left(\omega_i - \omega_{i-1} \right)^2 \cdot t3_i \right] \cdot l_i$$

Mixed

$$I_\omega = 2.638 \cdot 10^9 \circ mm^6$$

$$I_{yz} := \sum_{i=1}^{rows(y)-1} \left[\left(y_{i-1} \right) \cdot \left(z_{i-1} \right) \cdot t1_i + \left[y_{i-1} \cdot \left(z_i - z_{i-1} \right) + z_{i-1} \cdot \left(y_i - y_{i-1} \right) \right] \cdot t2_i + \left(y_i - y_{i-1} \right) \cdot \left(z_i - z_{i-1} \right) \cdot t3_i \right] \cdot l_i$$

$$I_{yz} = -1.059 \cdot 10^{-10} \circ mm^4$$

$$I_{y\omega} := \sum_{i=1}^{rows(y)-1} \left[\left(y_{i-1} \right) \cdot \left(\omega_{i-1} \right) \cdot t1_i + \left[y_{i-1} \cdot \left(\omega_i - \omega_{i-1} \right) + \omega_{i-1} \cdot \left(y_i - y_{i-1} \right) \right] \cdot t2_i + \left(y_i - y_{i-1} \right) \cdot \left(\omega_i - \omega_{i-1} \right) \cdot t3_i \right] \cdot l_i$$

$$I_{y\omega} = -5.018 \cdot 10^7 \circ mm^5$$

$$I_{z\omega} := \sum_{i=1}^{rows(y)-1} \left[(z_{i-1}) \cdot (\omega_{i-1}) \cdot t1_i + \left[z_{i-1} \cdot (\omega_i - \omega_{i-1}) + \omega_{i-1} \cdot (z_i - z_{i-1}) \right] \cdot t2_i + (z_i - z_{i-1}) \cdot (\omega_i - \omega_{i-1}) \cdot t3_i \right] \cdot l_i$$

$$I_{z\omega} = 1.689 \cdot 10^7 \circ mm^5$$

$$Ia_i := \overrightarrow{\left[(tl_i + th_i) \cdot \left[(tl_i)^2 + (th_i)^2 \right] \cdot \frac{l_i}{48} \right]}$$

Influence of thickness

$$I_{yz} := I_{yz} + \sum_{i=1}^{rows(y)-1} \frac{Ia_i \cdot (y_i - y_{i-1}) \cdot (z_i - z_{i-1})}{(l_i)^2}$$

$$I_{yz} = -1.061 \cdot 10^{-10} \circ mm^4$$

$$I_z := I_z + \sum_{i=1}^{rows(y)-1} \frac{Ia_i \cdot (z_i - z_{i-1})^2}{(l_i)^2}$$

$$I_z = 4.56 \cdot 10^6 \circ mm^4$$

$$I_y := I_y + \sum_{i=1}^{rows(y)-1} \frac{Ia_i \cdot (y_i - y_{i-1})^2}{(l_i)^2}$$

$$I_y = 3.413 \cdot 10^6 \circ mm^4$$

$$I_t := \sum_{i=1}^{rows(y)-1} (tl_i + th_i) \cdot \left[(tl_i)^2 + (th_i)^2 \right] \cdot \frac{l_i}{12} \cdot 1.05$$

$$I_t = 8.602 \cdot 10^4 \circ mm^4$$

$$z_{gc} := \frac{S_y}{A} \qquad z_{gc} = 6.274 \circ mm \qquad I_y := I_y - A \cdot z_{gc}^2 \qquad I_y = 3.259 \cdot 10^6 \circ mm^4$$

$$y_{gc} := \frac{S_z}{A} \qquad y_{gc} = 3.028 \cdot 10^{-15} \circ mm \qquad I_z := I_z - A \cdot y_{gc}^2 \qquad I_z = 4.56 \cdot 10^6 \circ mm^4$$

$$I_{yz} := I_{yz} - \frac{S_y \cdot S_z}{A} \quad I_{yz} = -1.805 \cdot 10^{-10} \circ mm^4 \quad I_{y\omega} := I_{y\omega} - \frac{S_z \cdot S_\omega}{A} \quad I_{y\omega} = -5.018 \cdot 10^7 \circ mm^5$$

$$I_\omega := I_\omega - \frac{S_\omega^2}{A} \quad I_\omega = 7.875 \cdot 10^8 \circ mm^6 \quad I_{z\omega} := I_{z\omega} - \frac{S_y \cdot S_\omega}{A} \quad I_{z\omega} = -3.309 \cdot 10^{-9} \circ mm^5$$

Principal axis

$$\alpha := if\left[\left| I_z - I_y \right| < (I_y + I_z) \cdot 10^{-9}, 0, \frac{1}{2} \cdot atan\left(\frac{2 \cdot I_{yz}}{I_z - I_y} \right) \right]$$

$$\alpha \cdot \frac{180}{\pi} = -7.944 \cdot 10^{-15}$$

$$I_\xi := \frac{1}{2} \cdot \left[I_y + I_z + \sqrt{(I_z - I_y)^2 + 4 \cdot I_{yz}^2} \right]$$

$$I_\xi = 4.56 \cdot 10^6 \circ mm^4$$

$$I_\eta := \frac{1}{2} \cdot \left[I_y + I_z - \sqrt{(I_z - I_y)^2 + 4 \cdot I_{yz}^2} \right]$$

$$I_\eta = 3.259 \cdot 10^6 \circ mm^4$$

Shear centre	$y_{sc} := \dfrac{I_{z\omega} \cdot I_z - I_{y\omega} \cdot I_{yz}}{I_y \cdot I_z - I_{yz}^2}$	$z_{sc} := \dfrac{-I_{y\omega} \cdot I_y + I_{z\omega} \cdot I_{yz}}{I_y \cdot I_z - I_{yz}^2}$	$y_{sc} = -1.625 \cdot 10^{-15} \circ mm$

$$z_{sc} = 11.005 \circ mm$$

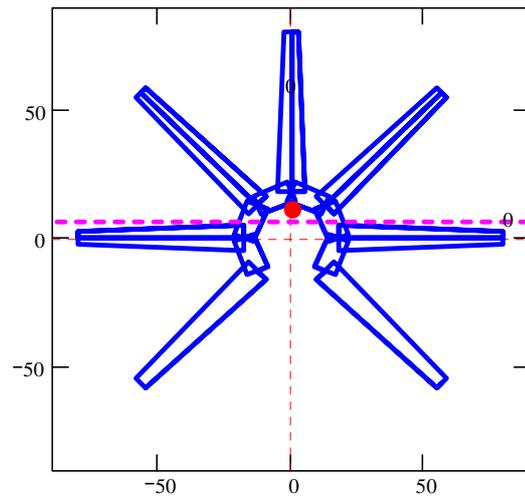Warping constant $\qquad I_w := I_\omega + z_{sc} \cdot I_{y\omega} - y_{sc} \cdot I_{z\omega}$ $\qquad\qquad I_w = 2.353 \cdot 10^8 \circ mm^6$

Polar radius gyration $\qquad i_p := \sqrt{\dfrac{I_y + I_z}{A} + \left(y_{sc} - y_{gc}\right)^2 + \left(z_{sc} - z_{gc}\right)^2}$ $\qquad i_p = 44.932 \circ mm$

Dashed line = gravity centre

Point = shear centre

Axial force resistance, flexural-torsional buckling

Buckling length $\quad l := L \qquad\qquad\qquad l = 1200 \circ mm$

Reference buckling loads $\qquad N_{Ey} := \dfrac{\pi^2 \cdot E \cdot I_y}{l^2} \qquad\qquad N_{Ez} := \dfrac{\pi^2 \cdot E \cdot I_z}{l^2} \qquad\qquad N_T := \left(G \cdot I_t + \dfrac{\pi^2 \cdot E \cdot I_\omega}{l^2}\right) \cdot \dfrac{1}{i_p^2}$

$\qquad N_{Ey} = 1.56 \cdot 10^3 \circ kN \qquad N_{Ez} = 2.19 \cdot 10^3 \circ kN \qquad N_T = 1.34 \cdot 10^3 \circ kN \qquad$ Guess: $\quad N := 0.2 \cdot N_{Ez}$

Given $\qquad \left(N_{Ey} - N\right) \cdot \left(N_{Ez} - N\right) \cdot \left(N_T - N\right) \cdot i_p^2 - \left(z_{sc} - z_{gc}\right)^2 \cdot N^2 \cdot \left(N_{Ey} - N\right) - \left(y_{sc} - y_{gc}\right)^2 \cdot N^2 \cdot \left(N_{Ez} - N\right) = 0$

Solution $\qquad N_{cr} := minerr(N) \qquad N_{cr} = 1.32 \cdot 10^3 \circ kN$

Cross-section entirely of radiating outstands, no welding, unsymmetical section

Table 5.5 and 5.8
$\alpha := 0.2$ $\lambda_1 := 0.6$ $k_2 := 1$

$\left(\lambda_c = \lambda_{bar}\right)$ $\lambda_c := \sqrt{\dfrac{f_o \cdot A}{N_{cr}}}$ $\lambda_c = 0.945$

(5.37) $\phi := 0.5 \cdot \left[1 + \alpha \cdot \left(\lambda_c - \lambda_1\right) + \lambda_c^2\right]$ $\chi := \dfrac{1}{\phi + \sqrt{\phi^2 - \lambda_c^2}}$ $\phi = 0.981$ $\chi = 0.804$

Table 5.5 $\psi := \dfrac{\left|min\left(z - z_{gc}\right)\right| - \left|max\left(z - z_{gc}\right)\right|}{\left|min\left(z - z_{gc}\right)\right| + \left|max\left(z - z_{gc}\right)\right|}$ $min\left(z - z_{gc}\right) = -62.8 \cdot mm$ $\psi = -0.08$

$max\left(z - z_{gc}\right) = 73.7 \cdot mm$

Table 5.5 $k_1 := 1 - 2.4 \cdot \psi^2 \cdot \dfrac{\lambda_c^2}{\left(1 + \lambda_c^2\right) \cdot \left(1 + \lambda_c\right)^2}$ $k_1 = 0.998$

5.8.3 (1) $N_{b.Rd} := \chi \cdot k_1 \cdot k_2 \cdot \dfrac{f_o}{\gamma_{M1}} \cdot A$ $N_{b.Rd} = 942.32 \cdot kN$