TALAT Lecture 2301

Design of Members

Axial Force

Example 5.4 : Axial force resistance of channel cross section

7 pages

Advanced Level

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Example 5.4. Axial force resistance of channel cross section

Width/depth	$b := 80 \cdot mm$	$h := 100 \cdot mm$	$f_o := 300 \cdot MPa$	$kN \equiv 1000 \cdot newton$
Inner thickness/outstand	$ti := 8 \cdot mm$	$c := 25 \cdot mm$	$E := 70000 \cdot MPa$ $\quad G := \dfrac{E}{2.6}$	$MPa \equiv 10^6 \cdot Pa$
Outer/bottom thickness	$to := 3.5 \cdot mm$	$tu := 4 \cdot mm$	$\gamma_{M1} := 1.0$	
Length	$L := 1200 \cdot mm$			

$i := 0 .. 8 \qquad k := 1 .. 3$

Nodes no., co-ordinates, thickness

$tl_i := to \qquad th_i := to \qquad y_k := 0.5 \cdot h \qquad z_i := b$

$th_1 := ti \qquad tl_8 := ti \qquad y_{k+4} := -0.5 \cdot h \quad z_{k+2} := 0 \cdot mm$

$th_4 := tu \qquad tl_4 := tu \qquad y_4 := 0 \cdot mm \qquad z_2 := 0.5 \cdot z_1$

$th_5 := th_4 \qquad tl_5 := tl_4 \qquad y_0 := 0.5 \cdot h - c \quad z_6 := 0.5 \cdot z_1$

$\qquad\qquad\qquad\qquad\qquad y_8 := -y_0 \qquad z_1 := z_1 - 0.5 \cdot (ti - to)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad z_7 := z_1$
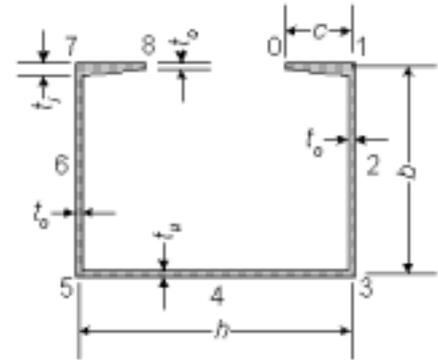
$i =$	$\dfrac{y_i}{mm} =$	$\dfrac{z_i}{mm} =$	$\dfrac{tl_i}{mm} =$	$\dfrac{th_i}{mm} =$
0	25	80	3.5	3.5
1	50	77.75	3.5	8
2	50	40	3.5	3.5
3	50	0	3.5	3.5
4	0	0	4	4
5	-50	0	4	4
6	-50	40	3.5	3.5
7	-50	77.75	3.5	3.5
8	-25	80	8	3.5

Nodes $\qquad i := 1 .. rows(y) - 1$

Cross-section constants, varying thickness

Sectorial co-ordinates $\qquad \omega_0 := 0 \cdot mm^2$

$$\omega_{0_i} := y_{i-1} \cdot z_i - y_i \cdot z_{i-1} \qquad\qquad \omega_i := \omega_{i-1} + \omega_{0_i}$$

Thicknesses to simplify expressions $\qquad Dt_i := th_i - tl_i \qquad t1_i := tl_i + \dfrac{Dt_i}{2} \qquad t2_i := \dfrac{tl_i}{2} + \dfrac{Dt_i}{3} \qquad t3_i := \dfrac{tl_i}{3} + \dfrac{Dt_i}{4}$

Length of elements $\qquad l_i := \overrightarrow{\left[\sqrt{\left(y_i - y_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2} \right]}$

Area $\qquad A := \displaystyle\sum_{i=1}^{rows(y)-1} \dfrac{tl_i + th_i}{2} \cdot l_i \qquad\qquad\qquad A = 1.233 \cdot 10^3 \cdot mm^2$

First moments of area $\qquad S_z := \displaystyle\sum_{i=1}^{rows(y)-1} \left[y_{i-1} \cdot t1_i + \left(y_i - y_{i-1}\right) \cdot t2_i \right] \cdot l_i \qquad S_z = -8.47 \cdot 10^{-13} \cdot mm^3$
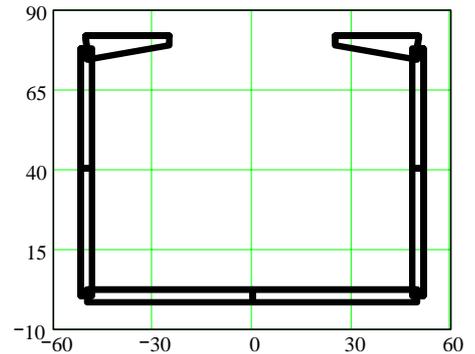
$$S_y := \sum_{i=1}^{rows(y)-1} \left[z_{i-1} \cdot t1_i + (z_i - z_{i-1}) \cdot t2_i \right] \cdot l_i \qquad\qquad S_y = 4.388 \cdot 10^4 \circ mm^3$$

$$S_\omega := \sum_{i=1}^{rows(y)-1} \left[\omega_{i-1} \cdot t1_i + (\omega_i - \omega_{i-1}) \cdot t2_i \right] \cdot l_i \qquad\qquad S_\omega = -7.328 \cdot 10^6 \circ mm^4$$

Second moments of area

$$I_z := \sum_{i=1}^{rows(y)-1} \left[(y_{i-1})^2 \cdot t1_i + 2 \cdot y_{i-1} \cdot (y_i - y_{i-1}) \cdot t2_i + (y_i - y_{i-1})^2 \cdot t3_i \right] \cdot l_i$$

$$I_y := \sum_{i=1}^{rows(y)-1} \left[(z_{i-1})^2 \cdot t1_i + 2 \cdot z_{i-1} \cdot (z_i - z_{i-1}) \cdot t2_i + (z_i - z_{i-1})^2 \cdot t3_i \right] \cdot l_i$$

$$I_\omega := \sum_{i=1}^{rows(y)-1} \left[(\omega_{i-1})^2 \cdot t1_i + 2 \cdot \omega_{i-1} \cdot (\omega_i - \omega_{i-1}) \cdot t2_i + (\omega_i - \omega_{i-1})^2 \cdot t3_i \right] \cdot l_i$$

Mixed

$$I_{yz} := \sum_{i=1}^{rows(y)-1} \left[(y_{i-1}) \cdot (z_{i-1}) \cdot t1_i + \left[y_{i-1} \cdot (z_i - z_{i-1}) + z_{i-1} \cdot (y_i - y_{i-1}) \right] \cdot t2_i + (y_i - y_{i-1}) \cdot (z_i - z_{i-1}) \cdot t3_i \right] \cdot l_i$$

$$I_{y\omega} := \sum_{i=1}^{rows(y)-1} \left[(y_{i-1}) \cdot (\omega_{i-1}) \cdot t1_i + \left[y_{i-1} \cdot (\omega_i - \omega_{i-1}) + \omega_{i-1} \cdot (y_i - y_{i-1}) \right] \cdot t2_i + (y_i - y_{i-1}) \cdot (\omega_i - \omega_{i-1}) \cdot t3_i \right] \cdot l_i$$

$$I_{z\omega} := \sum_{i=1}^{rows(y)-1} \left[(z_{i-1}) \cdot (\omega_{i-1}) \cdot t1_i + \left[z_{i-1} \cdot (\omega_i - \omega_{i-1}) + \omega_{i-1} \cdot (z_i - z_{i-1}) \right] \cdot t2_i + (z_i - z_{i-1}) \cdot (\omega_i - \omega_{i-1}) \cdot t3_i \right] \cdot l_i$$

Influence of thickness

$$Ia_i := \overrightarrow{\left[(tl_i + th_i) \cdot \left[(tl_i)^2 + (th_i)^2 \right] \cdot \frac{l_i}{48} \right]} \qquad I_{yz} := I_{yz} + \sum_{i=1}^{rows(y)-1} \frac{Ia_i \cdot (y_i - y_{i-1}) \cdot (z_i - z_{i-1})}{(l_i)^2}$$

$$I_z := I_z + \sum_{i=1}^{rows(y)-1} \frac{Ia_i \cdot (z_i - z_{i-1})^2}{(l_i)^2} \qquad\qquad I_y := I_y + \sum_{i=1}^{rows(y)-1} \frac{Ia_i \cdot (y_i - y_{i-1})^2}{(l_i)^2}$$

$$I_t := \sum_{i=1}^{rows(y)-1} (tl_i + th_i) \cdot \left[(tl_i)^2 + (th_i)^2 \right] \cdot \frac{l_i}{12} \cdot 1.05 \qquad\qquad I_t = 8.425 \cdot 10^3 \circ mm^4$$

$$z_{gc} := \frac{S_y}{A} \qquad\qquad z_{gc} = 35.593 \circ mm \qquad\qquad I_y := I_y - A \cdot z_{gc}^2 \qquad\qquad I_y = 1.325 \cdot 10^6 \circ mm^4$$

$$y_{gc} := \frac{S_z}{A} \qquad\qquad y_{gc} = 0 \circ mm \qquad\qquad I_z := I_z - A \cdot y_{gc}^2 \qquad\qquad I_z = 2.151 \cdot 10^6 \circ mm^4$$

$$I_{yz} := I_{yz} - \frac{S_y \cdot S_z}{A} \qquad I_{yz} = -1.816 \cdot 10^{-10} \circ mm^4 \qquad I_{y\omega} := I_{y\omega} - \frac{S_z \cdot S_\omega}{A} \qquad I_{y\omega} = 1.057 \cdot 10^8 \circ mm^5$$

$$I_\omega := I_\omega - \frac{S_\omega^2}{A} \qquad I_\omega = 9.438 \cdot 10^9 \circ mm^6 \qquad I_{z\omega} := I_{z\omega} - \frac{S_y \cdot S_\omega}{A} \qquad I_{z\omega} = 5.294 \cdot 10^{-8} \circ mm^5$$

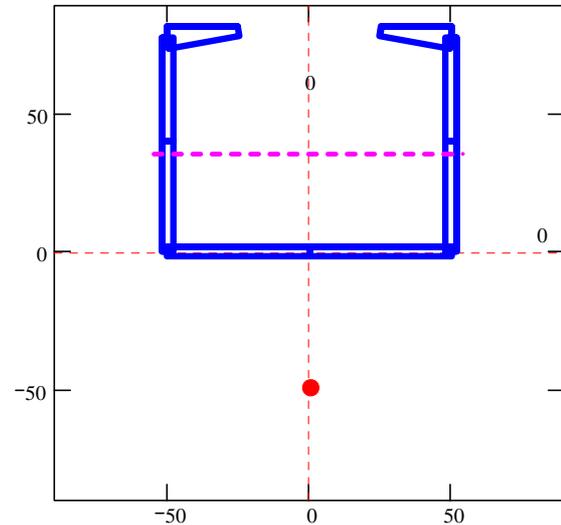| Shear centre | $y_{sc} := \dfrac{I_{z\omega} \cdot I_z - I_{y\omega} \cdot I_{yz}}{I_y \cdot I_z - I_{yz}^2}$ | $z_{sc} := \dfrac{-I_{y\omega} \cdot I_y + I_{z\omega} \cdot I_{yz}}{I_y \cdot I_z - I_{yz}^2}$ | $y_{sc} = 4.668 \cdot 10^{-14} \circ mm$ |

$z_{sc} = -49.159 \circ mm$

Warping constant

$$I_w := I_\omega + z_{sc} \cdot I_{y\omega} - y_{sc} \cdot I_{z\omega}$$

$I_w = 4.24 \cdot 10^9 \circ mm^6$

Polar radius gyration

$$i_p := \sqrt{\frac{I_y + I_z}{A} + \left(y_{sc} - y_{gc}\right)^2 + \left(z_{sc} - z_{gc}\right)^2}$$

$i_p = 100 \circ mm$

Buckling of stiffened flanges

| 5.4.5 | **Local buckling, internal elements** | $i := 3, 5 .. 7$ |

$$\varepsilon := \sqrt{\frac{250 \cdot MPa}{f_o}} \qquad \varepsilon = 0.913 \qquad \beta_i := \left[\sqrt{\left(y_i - y_{i-2}\right)^2 + \left(z_i - z_{i-2}\right)^2} \cdot \frac{2}{tl_{i-1} + th_i}\right]$$

5.4.5 (3) c)
heat-treated
unwelded

$$\rho_{c_i} := if\left[\frac{\beta_i}{\varepsilon} \le 22, 1.0, \frac{32}{\left(\frac{\beta_i}{\varepsilon}\right)} - \frac{220}{\left(\frac{\beta_i}{\varepsilon}\right)^2}\right] \qquad t_{efl_i} := \rho_{c_i} \cdot tl_i \qquad t_{efh_i} := \rho_{c_i} \cdot th_i$$

$$t_{efl_{i-1}} := \rho_{c_i} \cdot tl_i \qquad t_{efh_{i-1}} := \rho_{c_i} \cdot th_i$$

$\beta_i =$	$\rho_{c_i} =$	$\dfrac{t_{efl_i}}{mm} =$	$\dfrac{t_{efh_i}}{mm} =$
22.214	0.943	3.302	3.302
25	0.875	3.501	3.501
22.214	0.943	3.302	3.302

| 5.4.5 | **Local buckling, outstand elements** | $i := 1, 8 .. 8$ |

$$t_{fic_i} := if\left[tl_i > th_i, \left[\left(tl_i\right)^3 \cdot th_i\right]^{\frac{1}{4}}, \left[\left(th_i\right)^3 \cdot tl_i\right]^{\frac{1}{4}}\right] \qquad \beta_i := \left[\sqrt{\left(y_i - y_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2} \cdot \frac{1}{t_{fic_i}}\right]$$

5.4.5 (3) c)
heat-treated
unwelded

$$\rho_{c_i} := \text{if}\left[\frac{\beta_i}{\varepsilon} \le 6, 1.0, \frac{10}{\left(\frac{\beta_i}{\varepsilon}\right)} - \frac{24}{\left(\frac{\beta_i}{\varepsilon}\right)^2}\right] \qquad t_{efl_i} := \rho_{c_i} \cdot tl_i \qquad t_{efh_i} := \rho_{c_i} \cdot th_i$$

$\beta_i =$	$\rho_{c} =$	$\dfrac{t_{fic_i}}{mm} =$	$\dfrac{t_{efl_i}}{mm} =$	$\dfrac{t_{efh_i}}{mm} =$
3.858	1	6.506	3.5	8
3.858	1	6.506	8	3.5

Reference: Pekoz, T., Flanges of nonuniform thickness.
Aluminium Design Workshop, Cornell University Oct. 1997

Buckling of edge stiffener $\qquad i := 1..2 \qquad\qquad z_2 = 40 \cdot mm$

**Effective
area**

$$t1_i := t_{efl_i} + \frac{t_{efh_i} - t_{efl_i}}{2} \qquad\qquad t2_i := \frac{t_{efl_i}}{2} + \frac{t_{efh_i} - t_{efl_i}}{3}$$
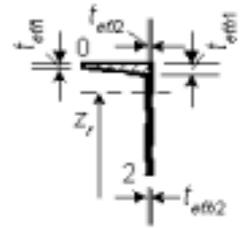
**Length of
elements**

$$l_i := \overrightarrow{\left[\sqrt{\left(y_i - y_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2}\right]}$$

Area

$$A_r := \sum_{i=1}^{2} \frac{t_{efl_i} + t_{efh_i}}{2} \cdot l_i \qquad\qquad A_r = 269 \cdot mm^2$$

**Centre of
gravity**

$$z_r := \sum_{i=1}^{2} \left[z_{i-1} \cdot t1_i + \left(z_i - z_{i-1}\right) \cdot t2_i\right] \cdot l_i \cdot \frac{1}{A_r} \qquad z_r = 69.528 \cdot mm$$

Second moment
of area

$$i := 1..2 \qquad z_2 := z_1 - 15 \cdot \left(tl_2 + th_2\right) \cdot 0.5 \qquad z_6 := z_2 \qquad z_2 = 25.25 \cdot mm$$

$$Dt_i := th_i - tl_i \qquad t1_i := tl_i + \frac{Dt_i}{2} \qquad t2_i := \frac{tl_i}{2} + \frac{Dt_i}{3} \qquad t3_i := \frac{tl_i}{3} + \frac{Dt_i}{4}$$
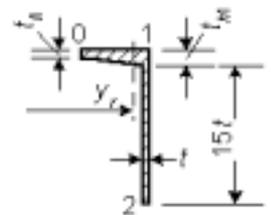
**Length of
elements**

$$l_i := \overrightarrow{\left[\sqrt{\left(y_i - y_{i-1}\right)^2 + \left(z_i - z_{i-1}\right)^2}\right]}$$

Area

$$A_{r1} := \sum_{i=1}^{2} \frac{tl_i + th_i}{2} \cdot l_i \qquad\qquad A_{r1} = 328.1 \cdot mm^2$$

**Centre of
gravity**

$$y_r := \sum_{i=1}^{2} \left[y_{i-1} \cdot t1_i + \left(y_i - y_{i-1}\right) \cdot t2_i\right] \cdot l_i \cdot \frac{1}{A_{r1}} \qquad y_r = 45.218 \cdot mm$$

<table>
<tr><td>Second moments of area</td></tr>
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Second
moments
of area

$$I_r := \sum_{i=1}^{2} \left[(y_{i-1})^2 \cdot t1_i + 2 \cdot y_{i-1} \cdot (y_i - y_{i-1}) \cdot t2_i + (y_i - y_{i-1})^2 \cdot t3_i \right] \cdot l_i - y_r^2 \cdot A_{r1}$$
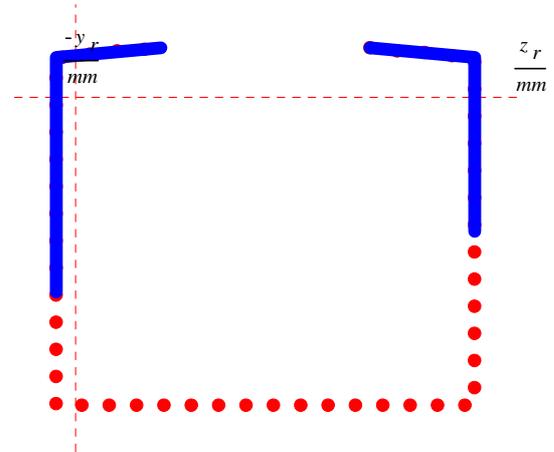
$$z_2 := z_1 \cdot 0.5$$

$$z_6 := z_2 \qquad k := 6..8 \qquad i := 0..2 \qquad j := 0..8$$

$$I_r = 1.668 \cdot 10^4 \circ mm^4$$

Left: Cross section for second moment of area I_r based on t

Right: Effective cross section area A_r based on t_{ef}

$$s_1 := y_3 - y_5 \qquad b_1 := z_r$$

$$b_1 = 69.528 \circ mm$$

$$s_1 = 100 \circ mm$$

$$N_{r.cr} := 1.05 \cdot E \cdot \sqrt{\frac{I_r \cdot (th_3)^3}{b_1^3 \cdot \left[1 + \frac{1.5 \cdot s_1 \cdot (th_4)^3}{b_1 \cdot (th_3)^3} \right]}}$$

$$N_{r.cr} = 52.195 \circ kN$$

Table 5.5
and 5.8

$$\alpha := 0.2 \qquad \lambda_1 := 0.6 \qquad \lambda_c := \sqrt{\frac{f_o \cdot A_r}{N_{r.cr}}} \qquad \lambda_{\bar{c}} \; 1.243$$

(5.37)

$$\phi := 0.5 \cdot \left[1 + \alpha \cdot (\lambda_c - \lambda_1) + \lambda_c^2 \right] \qquad \chi_{\bar{r}} = \frac{1}{\phi + \sqrt{\phi^2 - \lambda_c^2}} \qquad \chi_r := if(\chi_r > 1, 1, \chi_r)$$

$$\phi = 1.337$$

$$\chi_{\bar{r}} \; 0.547$$

$$i := 1..2 \qquad t_{efl_i} := \chi_r \cdot t_{efl_i} \qquad t_{efh_i} := \chi_r \cdot t_{efh_i}$$

$$i := 7..8 \qquad t_{efl_i} := \chi_r \cdot t_{efl_i} \qquad t_{efh_i} := \chi_r \cdot t_{efh_i}$$

$$i := 1..8$$

Length of
elements

$$l_i := \left[\sqrt{(y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \right]$$

Area

$$A_{eff} := \sum_{i=1}^{8} \frac{t_{efl_i} + t_{efh_i}}{2} \cdot l_i \qquad A_{eff} = 904.9 \circ mm^2$$

$\dfrac{t_{efl_i}}{mm} =$	$\dfrac{t_{efh_i}}{mm} =$	$\dfrac{l_i}{mm} =$
1.913	4.372	25.101
1.805	1.805	38.875
3.302	3.302	38.875
3.501	3.501	50
3.501	3.501	50
3.302	3.302	38.875
1.805	1.805	38.875
4.372	1.913	25.101

Axial force resistance, flexural-torsional buckling

Buckling length $l := L$ $\qquad l = 1200 \circ mm$

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Reference buckling loads	$N_{Ey} := \dfrac{\pi^2 \cdot E \cdot I_y}{l^2}$	$N_{Ez} := \dfrac{\pi^2 \cdot E \cdot I_z}{l^2}$	$N_T := \left(G \cdot I_t + \dfrac{\pi^2 \cdot E \cdot I_\omega}{l^2} \right) \cdot \dfrac{1}{i_p^{\,2}}$

$$N_{Ey} = 635.91 \circ kN \qquad N_{Ez} = 1.03 \cdot 10^3 \circ kN \qquad N_T = 475.37 \circ kN \qquad \text{Guess:} \quad N := 0.2 \cdot N_{Ez}$$

Given
$$\left(N_{Ey} - N\right) \cdot \left(N_{Ez} - N\right) \cdot \left(N_T - N\right) \cdot i_p^{\,2} - \left(z_{sc} - z_{gc}\right)^2 \cdot N^2 \cdot \left(N_{Ey} - N\right) - \left(y_{sc} - y_{gc}\right)^2 \cdot N^2 \cdot \left(N_{Ez} - N\right) = 0$$

Solution $\qquad N_{cr} := minerr(N) \qquad N_{cr} = 348.11 \circ kN$

General cross-section

Table 5.5 and 5.8 $\qquad \alpha := 0.35 \qquad \lambda_1 := 0.4 \qquad k_2 := 1$

$\left(\lambda_c = \lambda_{bar}\right) \quad \lambda_c := \sqrt{\dfrac{f_o \cdot A_{eff}}{N_{cr}}} \qquad\qquad \lambda_c = 0.883$

(5.37) $\qquad \phi := 0.5 \cdot \left[1 + \alpha \cdot \left(\lambda_c - \lambda_1 \right) + \lambda_c^{\,2} \right] \chi := \dfrac{1}{\phi + \sqrt{\phi^2 - \lambda_c^{\,2}}} \qquad \phi = 0.974 \qquad \chi = 0.721$

Table 5.5 $\qquad \psi := \dfrac{\left| min\left(z - z_{gc}\right) \right| - \left| max\left(z - z_{gc}\right) \right|}{\left| min\left(z - z_{gc}\right) \right| + \left| max\left(z - z_{gc}\right) \right|}$ $\qquad min\left(z - z_{gc}\right) = -35.6 \circ mm \qquad \psi = -0.11$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad max\left(z - z_{gc}\right) = 44.4 \circ mm$

Table 5.5 $\qquad k_1 := 1 - 2.4 \cdot \psi^2 \cdot \dfrac{\lambda_c^{\,2}}{\left(1 + \lambda_c^{\,2}\right) \cdot \left(1 + \lambda_c\right)^2}$ $\qquad\qquad\qquad\qquad\qquad\qquad k_1 = 0.996$

5.8.3 (1) $\qquad N_{b.Rd} := \chi \cdot k_1 \cdot k_2 \cdot \dfrac{f_o}{\gamma_{M1}} \cdot A \qquad\qquad\qquad\qquad\qquad\qquad N_{b.Rd} = 265.8 \circ kN$