

**TALAT Lecture 2301**

**Design of Members**

**Axial Force**

**Example 5.7 : Axial force resistance of orthotropic  
double-skin plate**

9 pages

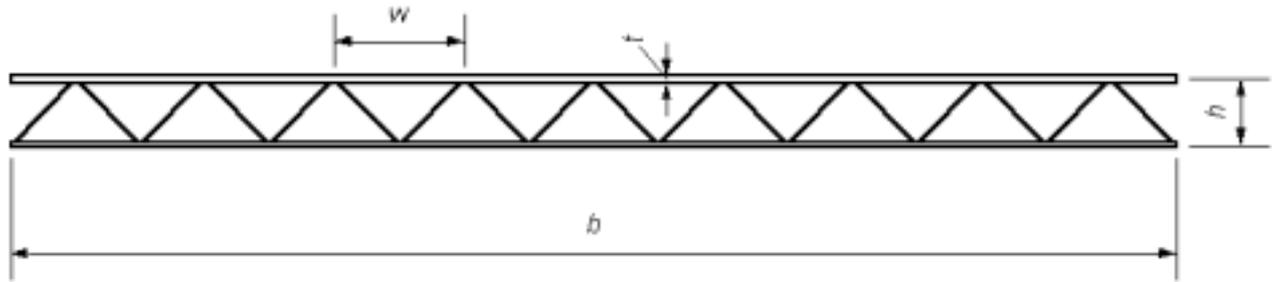
Advanced Level

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## Example 5.7. Axial force resistance of orthotropic double-skin plate

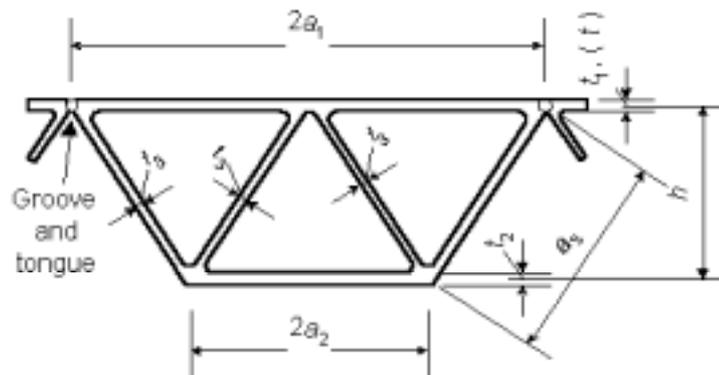


### Input (highlighted)

$N \equiv \text{newton}$   
 $kN \equiv 1000 \cdot N$   
 $MPa \equiv 10^6 \cdot Pa$

Plate thickness  $t := 5 \cdot \text{mm}$   $t_1 := t$   $f_o := 240 \cdot \text{MPa}$   
 Plate width  $b := 300000 \cdot \text{mm}$   $f_u := 260 \cdot \text{MPa}$   
 Plate length  $L := 5000 \cdot \text{mm}$   $E := 70000 \cdot \text{MPa}$   
 "Pitch" (2a or d)  $w := 160 \cdot \text{mm}$   $a := \frac{w}{2}$   $\gamma_{M1} := 1.1$   
 If heat-treated alloy, then  $ht = 1$  else  $ht = 0$   $cf := 0$   $ht := 1$

### a) Profiles with groove and tongue



Half bottom flange  $a_2 := 40 \cdot \text{mm}$   
 Thickness of bottom flange  $t_2 := 5 \cdot \text{mm}$   
 Profile depth  $h := 70 \cdot \text{mm}$   
 Web thickness  $t_3 := 5 \cdot \text{mm}$   
 Half width of trapezoidal stiffener at the top  $a_1 := 80 \cdot \text{mm}$   
 Number of webs  $n_w := 4$   
 Width of web  $a_3 := \sqrt{(a_1 - a_2)^2 + h^2}$   $a_3 = 80.6 \cdot \text{mm}$

## Local buckling

Internal elements

$$\beta_{i1} := \frac{a_1}{t_1} \quad \beta_{i2} := \frac{2 \cdot a_2}{t_2} \quad \beta_{i3} := \frac{a_3}{t_3}$$

$$\beta_i = \begin{bmatrix} 0 \\ 16 \\ 16 \\ 16.125 \end{bmatrix}$$

$$\beta := \max(\beta_i) \quad \beta = 16.1$$

[1] Tab. 5.1

$$\varepsilon := \sqrt{\frac{250 \text{ newton}}{f_o \text{ mm}^2}}$$

$$\beta_{\bar{1}} = 9 \cdot \varepsilon$$

$$\beta_{\bar{1}} = 9.186$$

$$\beta_{\bar{2}} = 13 \cdot \varepsilon$$

$$\beta_{\bar{2}} = 13.268$$

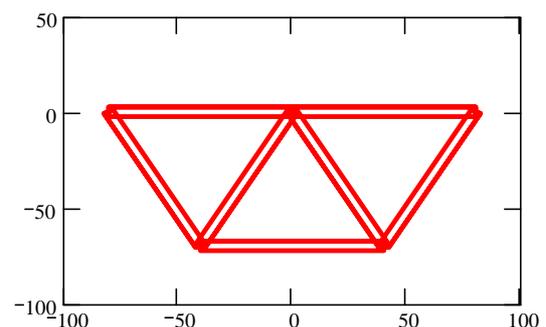
$$\beta_{\bar{3}} = 18 \cdot \varepsilon$$

$$\beta_{\bar{3}} = 18.371$$

$$\text{class}_i := \text{if}(\beta > \beta_{\bar{1}}, \text{if}(\beta > \beta_{\bar{2}}, \text{if}(\beta > \beta_{\bar{3}}, 4, 3), 2), 1)$$

$$\text{class}_i = 3$$

No reduction for local buckling



## Overall buckling, uniform compression

Cross sectional area

$$A := 2 \cdot t_1 \cdot a_1 + 2 \cdot t_2 \cdot a_2 + 2 \cdot t_3 \cdot a_3 \cdot \frac{n_w}{2}$$

$$A = 2.812 \cdot 10^3 \cdot \text{mm}^2$$

Gravity centre

$$e := \frac{2 \cdot t_2 \cdot a_2 \cdot h + 2 \cdot t_3 \cdot a_3 \cdot \frac{h}{2} \cdot \frac{n_w}{2}}{A}$$

$$e = 30.022 \cdot \text{mm}$$

Second moment of area

$$I_L := 2 \cdot t_2 \cdot a_2 \cdot h^2 + 2 \cdot t_3 \cdot a_3 \cdot \frac{h^2}{3} \cdot \frac{n_w}{2} - A \cdot e^2$$

$$I_L = 2.059 \cdot 10^6 \cdot \text{mm}^4$$

Approx. torsional constant

$$I_T := \frac{4 \cdot [h \cdot (a_1 + a_2)]^2}{\frac{2 \cdot a_1}{t_1} + \frac{2 \cdot a_2}{t_2} + 2 \cdot \frac{a_3}{t_3}}$$

$$I_T = 3.517 \cdot 10^6 \cdot \text{mm}^4$$

Rigidities of orthotropic plate

Table 5.10

$$B_x := \frac{E \cdot I_L}{2 \cdot a}$$

$$\nu := 0.3$$

$$B_x = 9.007 \cdot 10^8 \cdot \left( \frac{\text{N} \cdot \text{mm}^2}{\text{mm}} \right)$$

Table 5.10

$$B_y := 0.001 \cdot N \cdot \text{mm}$$

$$G := \frac{E}{2 \cdot (1 + \nu)}$$

$$B_y = 1 \cdot 10^{-3} \cdot \left( \frac{\text{N} \cdot \text{mm}^2}{\text{mm}} \right)$$

Table 5.10

$$H := \frac{G \cdot I_T}{2 \cdot a}$$

$$H = 5.918 \cdot 10^8 \cdot \left( \frac{\text{N} \cdot \text{mm}^2}{\text{mm}} \right)$$

Elastic buckling load

$$(5.77) \quad N_{cr} := \begin{cases} \frac{\pi^2}{b} \cdot \left[ \frac{B_x}{\left(\frac{L}{b}\right)^2} + 2 \cdot H + B_y \cdot \left(\frac{L}{b}\right)^2 \right] & \text{if } \frac{L}{b} < \sqrt[4]{\frac{B_x}{B_y}} \\ \frac{2 \cdot \pi^2}{b} \cdot \left( \sqrt{B_x \cdot B_y} + H \right) & \text{otherwise} \end{cases} \quad N_{cr} = 1.067 \cdot 10^5 \cdot kN$$

(5.78)

Buckling resistance

$$(5.69) \quad A_{ef} := A \quad A_{ef} = 2.812 \cdot 10^3 \cdot mm^2$$

$$\lambda_{\bar{c}} := \sqrt{\frac{A_{ef} \cdot f_o}{N_{cr}}} \quad \lambda_{\bar{c}} = 0.08$$

$$\alpha := \text{if}(ht > 0, 0.2, 0.32) \quad \lambda_{\bar{\sigma}} := \text{if}(ht > 0, 0.1, 0) \quad \alpha = 0.2$$

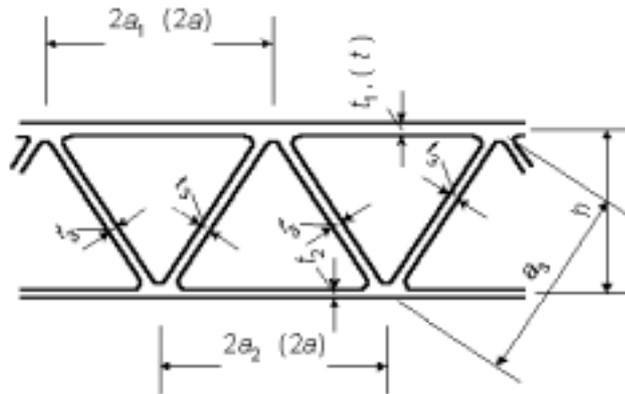
$$\phi := 0.5 \cdot \left[ 1 + \left[ \alpha \cdot (\lambda_{\bar{c}} - \lambda_{\bar{\sigma}}) + \lambda_{\bar{c}}^2 \right] \right] \quad \lambda_{\bar{\sigma}} = 0.1$$

$$\phi = 0.501$$

$$(5.33) \quad \chi_{\bar{c}} := \frac{1}{\phi + \sqrt{\phi^2 - \lambda_{\bar{c}}^2}} \quad \chi_{\bar{c}} = 1.004$$

$$(5.68) \quad N_{cRd} := A_{ef} \chi_{\bar{c}} \cdot \frac{f_o}{\gamma_{MI}} \quad \text{for one stiffener} \quad N_{cRd} = 616.164 \cdot kN$$

## b) Truss cross section



Half bottom flange  $a_2 := \frac{a}{2}$   $a_1 := \frac{a}{2}$   $a = 80 \cdot \text{mm}$

Stiffener depth  $h := 70 \cdot \text{mm}$   $a_1 = 40 \cdot \text{mm}$

Thickness of bottom flange  $t_2 := 5 \cdot \text{mm}$   $a_2 = 40 \cdot \text{mm}$

Web thickness  $t_3 := 5 \cdot \text{mm}$

Width of web  $a_3 := \sqrt{a_1^2 + h^2}$   $a_3 = 80.623 \cdot \text{mm}$

### Local buckling

Internal elements  $\beta_{1i} := \frac{2 \cdot a_1}{t_1}$   $\beta_{2i} := \frac{2 \cdot a_2}{t_2}$   $\beta_{3i} := \frac{a_3}{t_3}$   $\beta_i = \begin{bmatrix} 0 \\ 16 \\ 16 \\ 16.125 \end{bmatrix}$

$\beta := \max(\beta_i)$   $\beta = 16.1$

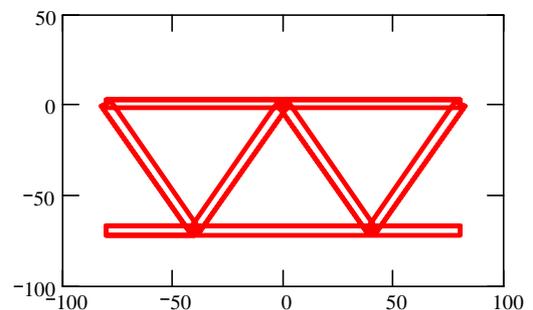
[1] Tab. 5.1  $\varepsilon := \frac{250 \cdot \text{newton}}{\sqrt{f_o} \cdot \text{mm}^2}$   $\beta_{\bar{1}} = 9 \cdot \varepsilon$   $\beta_{\bar{1}} = 9.186$

$\beta_{\bar{2}} = 13 \cdot \varepsilon$   $\beta_{\bar{2}} = 13.268$

$\beta_{\bar{3}} = 18 \cdot \varepsilon$   $\beta_{\bar{3}} = 18.371$

$\text{class}_i := \text{if}(\beta > \beta_{\bar{1}}, \text{if}(\beta > \beta_{\bar{2}}, \text{if}(\beta > \beta_{\bar{3}}, 4, 3), 2), 1)$   $\text{class}_i = 3$

No reduction for local buckling



## Overall buckling, uniform compression

Cross sectional area  $A := 2 \cdot t_1 \cdot a_1 + 2 \cdot t_2 \cdot a_2 + 2 \cdot t_3 \cdot a_3$   $A = 1.606 \cdot 10^3 \cdot \text{mm}^2$

Gravity centre  $e := \frac{2 \cdot t_2 \cdot a_2 \cdot h + 2 \cdot t_3 \cdot a_3 \cdot \frac{h}{2}}{A}$   $e = 35 \cdot \text{mm}$

Second moment of area  $I_L := 2 \cdot t_2 \cdot a_2 \cdot h^2 + 2 \cdot t_3 \cdot a_3 \cdot \frac{h^2}{3} - A \cdot e^2$   $I_L = 1.309 \cdot 10^6 \cdot \text{mm}^4$

Torsion constant  $I_T := \frac{4 \cdot [h \cdot (a_1 + a_2)]^2}{\frac{2 \cdot a_1}{t_1} + \frac{2 \cdot a_2}{t_2} + 2 \cdot \frac{a_3}{t_3}}$   $I_T = 1.952 \cdot 10^6 \cdot \text{mm}^4$

Orthotropic plate constant

Table 5.10  $B_x := \frac{E \cdot I_L}{2 \cdot a}$   $B_x = 5.728 \cdot 10^8 \cdot \left( \frac{\text{N} \cdot \text{mm}^2}{\text{mm}} \right)$

Table 5.10  $B_y := \frac{E \cdot t_1 \cdot t_2 \cdot h^2}{t_1 + t_2}$   $B_y = 8.575 \cdot 10^8 \cdot \left( \frac{\text{N} \cdot \text{mm}^2}{\text{mm}} \right)$

Table 5.10  $H := \frac{G \cdot I_T}{2 \cdot a}$   $H = 3.285 \cdot 10^8 \cdot \left( \frac{\text{N} \cdot \text{mm}^2}{\text{mm}} \right)$

Elastic buckling load

(5.77)  $N_{cr} := \left[ \frac{\pi^2}{b} \cdot \left[ \frac{B_x}{\left( \frac{L}{b} \right)^2} + 2 \cdot H + B_y \cdot \left( \frac{L}{b} \right)^2 \right] \right] \text{ if } \frac{L}{b} < \sqrt[4]{\frac{B_x}{B_y}}$   $N_{cr} = 6.786 \cdot 10^4 \cdot \text{kN}$

(5.78)  $\left[ \frac{2 \cdot \pi^2}{b} \cdot \left( \sqrt{B_x \cdot B_y} + H \right) \right] \text{ otherwise}$

Buckling resistance

$A_{ef} := A$   $A_{ef} = 1.606 \cdot 10^3 \cdot \text{mm}^2$

(5.69)  $\lambda_{\bar{c}} := \sqrt{\frac{A_{ef} \cdot f_o}{N_{cr}}}$   $\lambda_{\bar{c}} = 0.08$

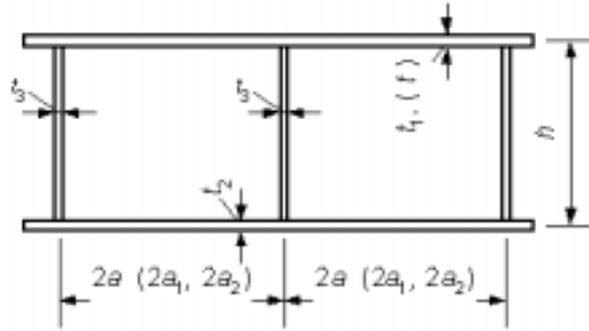
$\alpha := \text{if}(ht > 0, 0.2, 0.32)$   $\lambda_{\bar{\sigma}} := \text{if}(ht > 0, 0.1, 0)$   $\alpha = 0.2$

$\phi := 0.5 \cdot \left[ 1 + \left[ \alpha \cdot (\lambda_{\bar{c}} - \lambda_{\bar{\sigma}}) + \lambda_{\bar{c}}^2 \right] \right]$   $\lambda_{\bar{\sigma}} = 0.1$

(5.33)  $\chi_{\bar{c}} := \frac{1}{\phi + \sqrt{\phi^2 - \lambda_{\bar{c}}^2}}$   $\phi = 0.5$   
 $\chi_{\bar{c}} = 1.005$

(5.68)  $N_{cRd} := 2 \cdot A_{ef} \cdot \chi_{\bar{c}} \cdot \frac{f_o}{\gamma_{MI}}$  for two pitches  $N_{cRd} = 704.4 \cdot \text{kN}$

### c) Frame cross section



Half bottom flange	$a := 37.5 \cdot mm$	$a_1 := a$	
Stiffener depth	$h := 70 \cdot mm$	$a_2 := a$	
Thickness of top flange	$t_1 := 5 \cdot mm$		$a_1 = 37.5 \cdot mm$
Thickness of bottom flange	$t_2 := 5 \cdot mm$		$a_2 = 37.5 \cdot mm$
Web thickness	$t_3 := 5 \cdot mm$		
Width of web	$a_3 := h$		$a_3 = 70 \cdot mm$

#### Local buckling

Internal elements

$$\beta_{1^i} = \frac{2 \cdot a_1}{t_1} \quad \beta_{2^i} = \frac{2 \cdot a_2}{t_2} \quad \beta_{3^i} = \frac{a_3}{t_3}$$

$$\beta := \max(\beta_i) \quad \beta = 15$$

$$\beta_i = \begin{bmatrix} 0 \\ 15 \\ 15 \\ 14 \end{bmatrix}$$

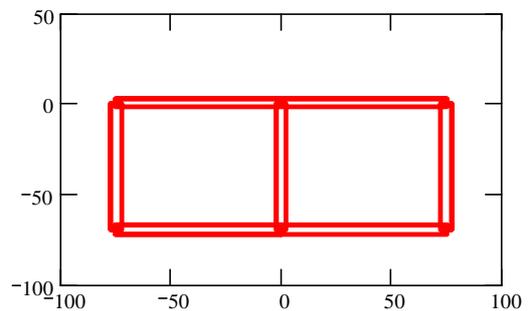
[1] Tab. 5.1

$$\varepsilon := \sqrt{\frac{250 \cdot \text{newton}}{f_o \cdot mm^2}}$$

$\beta_{\bar{1}} = 9 \cdot \varepsilon$	$\beta_{\bar{2}} = 13 \cdot \varepsilon$	$\beta_{\bar{3}} = 18 \cdot \varepsilon$
$\beta_{\bar{1}} = 9.186$	$\beta_{\bar{2}} = 13.268$	$\beta_{\bar{3}} = 18.371$

$$class_i := \text{if}(\beta > \beta_{\bar{1}}, \text{if}(\beta > \beta_{\bar{2}}, \text{if}(\beta > \beta_{\bar{3}}, 4, 3), 2), 1) \quad class_i = 3$$

No reduction for local buckling



## Overall buckling, uniform compression

Cross sectional area  $A := 2 \cdot t_1 \cdot a_2 + 2 \cdot t_2 \cdot a_2 + t_3 \cdot a_3$   $A = 1.1 \cdot 10^3 \cdot \text{mm}^2$

Gravity centre  $e := \frac{2 \cdot t_2 \cdot a_2 \cdot h + t_3 \cdot a_3 \cdot \frac{h}{2}}{A}$   $e = 35 \cdot \text{mm}$

Second moment of area  $I_L := 2 \cdot t_2 \cdot a_2 \cdot h^2 + t_3 \cdot a_3 \cdot \frac{h^2}{3} - A \cdot e^2$   $I_L = 1.062 \cdot 10^6 \cdot \text{mm}^4$

Torsion constant  $I_T := \frac{4 \cdot [h \cdot (a_1 + a_2)]^2}{\frac{2 \cdot a_1}{t_1} + \frac{2 \cdot a_2}{t_2} + 2 \cdot \frac{a_3}{t_3}}$   $I_T = 1.901 \cdot 10^6 \cdot \text{mm}^4$

Orthotropic plate constant

(5.80d)  $B_x := \frac{E \cdot I_L}{2 \cdot a}$   $B_x = 9.909 \cdot 10^8 \cdot \left( \frac{\text{N} \cdot \text{mm}^2}{\text{mm}} \right)$

(5.80a)  $B_y := \frac{E \cdot t_1^3}{12 \cdot (1 - \nu^2)} \cdot \frac{10 \cdot b^2}{32 \cdot a^2} \cdot \frac{a \cdot t_3^3 + \frac{a \cdot t_2^3 \cdot t_3^3}{t_1^3} + 6 \cdot h \cdot t_2^3}{a \cdot t_3^3 + 2 \cdot h \cdot (t_1^3 + t_2^3) + \frac{3 \cdot h^2 \cdot t_1^3 \cdot t_2^3}{a \cdot t_3^3}} \cdot \frac{t_1^2}{L^2}$

(5.80b)  $H := \frac{2 \cdot E}{3 \cdot \left(1 - \frac{t_3}{2 \cdot a}\right)} \cdot \left[ \frac{t_1^3}{1 + \frac{6 \cdot t_1}{2 \cdot a - t_3}} + \frac{t_2^3}{1 + \frac{6 \cdot t_2}{2 \cdot a - t_3}} \right]$   $B_y = 1.118 \cdot 10^7 \cdot \left( \frac{\text{N} \cdot \text{mm}^2}{\text{mm}} \right)$   
 $H = 8.75 \cdot 10^6 \cdot \left( \frac{\text{N} \cdot \text{mm}^2}{\text{mm}} \right)$

Elastic buckling load

(5.77)  $N_{cr} := \left\{ \frac{\pi^2}{b} \cdot \left[ \frac{B_x}{\left(\frac{L}{b}\right)^2} + 2 \cdot H + B_y \cdot \left(\frac{L}{b}\right)^2 \right] \right. \text{if } \frac{L}{b} < \sqrt{\frac{B_x}{B_y}}$   $N_{cr} = 1.174 \cdot 10^5 \cdot \text{kN}$

(5.78)  $\left. \frac{2 \cdot \pi^2}{b} \cdot \left( \sqrt{B_x \cdot B_y} + H \right) \right\} \text{otherwise}$   $\frac{2 \cdot \pi^2}{b} \cdot \left( \sqrt{B_x \cdot B_y} + H \right) = 7.501 \cdot \text{kN}$

Buckling resistance  $A_{ef} := A$   $A_{ef} = 1.1 \cdot 10^3 \cdot \text{mm}^2$

(5.69)  $\lambda_{\bar{c}} := \sqrt{\frac{A_{ef} \cdot f_o}{N_{cr}}}$   $\lambda_{\bar{c}} = 0.047$

$\alpha := \text{if}(ht > 0, 0.2, 0.32)$   $\lambda_{\bar{\sigma}} := \text{if}(ht > 0, 0.1, 0)$   $\alpha = 0.2$

$\phi := 0.5 \cdot \left[ 1 + \left[ \alpha \cdot (\lambda_{\bar{c}} - \lambda_{\bar{\sigma}}) + \lambda_{\bar{c}}^2 \right] \right]$   $\lambda_{\bar{\sigma}} = 0.1$

(5.33)  $\chi_{\bar{c}} := \frac{1}{\phi + \sqrt{\phi^2 - \lambda_{\bar{c}}^2}}$   $\phi = 0.496$   
 $\chi_{\bar{c}} = 1.011$

(5.68)  $N_{cRd} := 2 \cdot A_{ef} \cdot \chi_{\bar{c}} \cdot \frac{f_o}{\gamma_{MI}}$  for two pitches  $N_{cRd} = 485.112 \cdot \text{kN}$

## Summary

a)	$N_{Rd.a} = 616.2 \cdot kN$	$A_a = 2.812 \cdot 10^3 \cdot mm^2$	$\frac{N_{Rd.a}}{A_a} = 219.1 \cdot \frac{N}{mm^2}$
b)	$N_{Rd.b} = 704.4 \cdot kN$	$A_b = 3.212 \cdot 10^3 \cdot mm^2$	$\frac{N_{Rd.b}}{A_b} = 219.3 \cdot \frac{N}{mm^2}$
c)	$N_{Rd.c} = 485.1 \cdot kN$	$A_c = 2.2 \cdot 10^3 \cdot mm^2$	$\frac{N_{Rd.c}}{A_c} = 220.5 \cdot \frac{N}{mm^2}$