TALAT Lecture 2301

Design of Members

Shear Force

Example 6.1 – 6.6 : Shear resistance of webs without and with stiffeners

11 pages

Advanced Level

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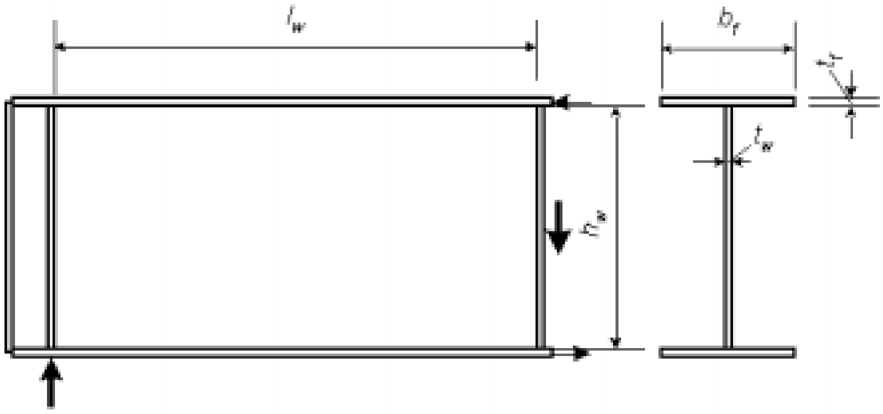
Example 6.1 - 6.6 Shear resistance of webs without and with stiffeners



Dimensions and strength of material — Data input in highlighted regions

Web depth	$h_w := 2000 \cdot mm$	$MPa \equiv 1000000 \cdot Pa$
Web thickness	$t_w := 15 \cdot mm$	$kN \equiv 1000 \cdot newton$
Total length of web panel	$l_w := 4000 \cdot mm$	$spv \equiv 2.4 \cdot \frac{kg}{(0.1 \cdot m)^3}$ (Mass)
0,2 proof strength of web plate material	$f_{ow} := 355 \cdot MPa$	
Ultimate strength of web plate material	$f_{uw} := 470 \cdot MPa$	
Elastic modulus	$E := 70000 \cdot MPa$	
Partial coefficient	$\gamma_{M1} := 1.1$	

Table 5.12 $\quad \eta := 0.4 + 0.2 \cdot \frac{f_{uw}}{f_{ow}} \quad \eta = 0.665$

1) No intermediate stiffener

If "rigid end post" then *endpost* = 1 else *endpost* = 0 $\quad endpost := 1$

Length of web panel $\quad a := l_w \quad a = 4000 \circ mm \quad \frac{a}{h_w} = 2$

Ref. to Eurocode 9

(5.97) $\quad k_\tau := if\left[\frac{a}{h_w} > 1.00, 5.34 + 4.00 \cdot \left(\frac{h_w}{a}\right)^2, 4.00 + 5.34 \cdot \left(\frac{h_w}{a}\right)^2\right] \quad k_\tau = 6.34$

(5.96) $\quad \lambda_w := \frac{0.81}{\sqrt{k_\tau}} \cdot \frac{h_w}{t_w} \cdot \sqrt{\frac{f_{ow}}{E}} \quad \lambda_w = 3.055$

Table 5.12 $\quad \rho_v := if\left(\lambda_w > 0.949, \frac{1.32}{1.66 + \lambda_w}, \frac{0.48}{\lambda_w}\right) \quad \rho_v = 0.28$

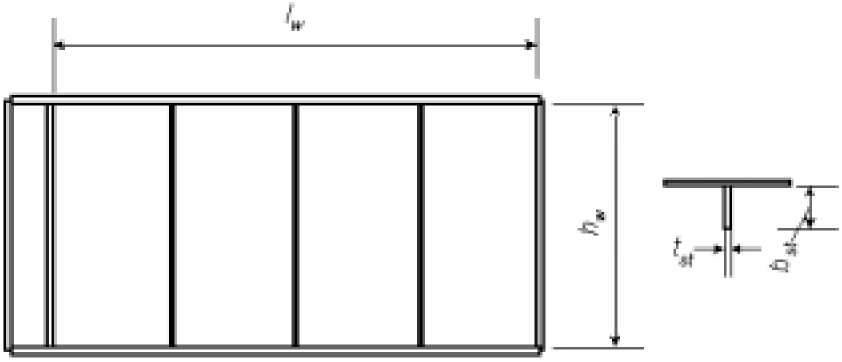
Table 5.12 $\quad \rho_v := if(\rho_v > \eta, \eta, \rho_v) \quad \rho_v = 0.28$

Table 5.12 $\quad \rho_v := if\left(endpost = 0, if\left(\rho_v > \frac{0.48}{\lambda_w}, \frac{0.48}{\lambda_w}, \rho_v\right), \rho_v\right) \quad \rho_v = 0.28$

(5.95) $\quad V_{w.Rd} := \rho_v \cdot t_w \cdot h_w \cdot \frac{f_{ow}}{\gamma_{M1}} \quad V_{w.Rd} = 2.711 \cdot 10^3 \circ kN$

Weight of web $\quad Mass_1 := (h_w \cdot t_w \cdot l_w) \cdot spv \quad Mass_1 = 288 \text{ kg} \quad V_{Rd_1} := V_{w.Rd}$

2) Equally spaced flexible transverse stiffeners



If "rigid end post" then *endpost* = 1 else *endpost* = 0 $\quad endpost := 1$

Number of stiffeners $\quad n_{st} := 3$

Single plate stiffener $\quad t_{st} := 15 \cdot mm$

$b_{st} := 120 \cdot mm \qquad \frac{b_{st}}{t_{st}} = 8$

Whole web panel. Buckling of stiffener

Figure 5.21 $\quad A_{st} := t_{st} \cdot b_{st} + 30 \cdot t_w^{\,2} \qquad A_{st} = 8.55 \cdot 10^3 \circ mm^2$

C.G. $\quad e_{st} := \frac{t_{st} \cdot b_{st}^{\,2}}{2 \cdot A_{st}} \qquad e_{st} = 12.632 \circ mm$

Second moment of area $\quad I_{st} := \frac{t_{st} \cdot b_{st}^{\,3}}{3} - A_{st} \cdot e_{st}^{\,2} \qquad I_{st} = 7.276 \cdot 10^6 \circ mm^4$

Panel length $\quad a := l_w \qquad a = 4 \cdot 10^3 \circ mm$

(5.99) $\quad k_{\tau\ st} := 9 \cdot \left(\frac{h_w}{a}\right)^2 \cdot \left(\frac{I_{st}}{t_w^{\,3} \cdot h_w}\right)^{\frac{3}{4}} \qquad k_{\tau\ st} = 2.38$

(5.99) $\quad k_{\tau\ stmin} := \frac{2.1}{t_w} \cdot \left(\frac{I_{st}}{h_w}\right)^{\frac{1}{3}} \qquad k_{\tau\ stmin} = 2.153$

$k_{\tau\ st} := if\left(k_{\tau\ st} < k_{\tau\ stmin}, k_{\tau\ stmin}, k_{\tau\ st}\right) \qquad k_{\tau\ st} = 2.38$

(5.97)

(5.98) $\quad k_{\tau} := if\left[\frac{a}{h_w} > 1.00, 5.34 + 4.00 \cdot \left(\frac{h_w}{a}\right)^2, 4.00 + 5.34 \cdot \left(\frac{h_w}{a}\right)^2\right] \qquad k_{\tau} = 6.34$

$k_{\tau} := k_{\tau} + k_{\tau\ st} \qquad k_{\tau} = 8.72$

(5.96) $\quad \lambda_w := \frac{0.81}{\sqrt{k_{\tau}}} \cdot \frac{h_w}{t_w} \cdot \sqrt{\frac{f_{ow}}{E}} \qquad \lambda_w = 2.605$

$\lambda_{wtot} := \lambda_w$

Sub panel

Length $a := \frac{l_w}{n_{st}+1}$ $a = 1\cdot 10^3\,mm$ $\frac{a}{h_w} = 0.5$

(5.97 and 5.98) $k_\tau := if\left[\frac{a}{h_w} > 1.00, 5.34 + 4.00\cdot\left(\frac{h_w}{a}\right)^2, 4.00 + 5.34\cdot\left(\frac{h_w}{a}\right)^2\right]$ $k_\tau = 25.36$

(5.96) $\lambda_w := \frac{0.81}{\sqrt{k_\tau}}\cdot\frac{h_w}{t_w}\cdot\sqrt{\frac{f_{ow}}{E}}$ $\lambda_{wsub} := \lambda_w$ $\lambda_{wsub} = 1.527$

$\lambda_{wtot} = 2.605$

The larger of the slenderness parameter λ_{wtot} for the total panel and λ_{wsub} for the sub panels is used. If $\lambda_{wtot} > \lambda_{wsub}$ then the stiffeners are flexible else the stiffeners are rigid.

$\lambda_w := if(\lambda_{wtot} > \lambda_{wsub}, \lambda_{wtot}, \lambda_{wsub})$ $\lambda_w = 2.605$

Table 5.12 $\rho_v := if\left(\lambda_w > 0.949, \frac{1.32}{1.66+\lambda_w}, \frac{0.48}{\lambda_w}\right)$ $\rho_v := if(\rho_v > \eta, \eta, \rho_v)$ $\rho_v = 0.31$

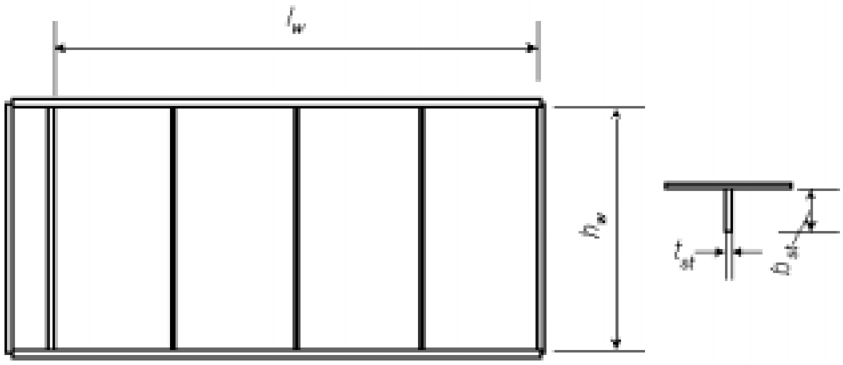
Table 5.12 $\rho_v := if\left(endpost = 0, if\left(\rho_v > \frac{0.48}{\lambda_w}, \frac{0.48}{\lambda_w}, \rho_v\right), \rho_v\right)$ $\rho_v = 0.31$

(5.95) $V_{wRd} := \rho_v\cdot t_w\cdot h_w\cdot\frac{f_{ow}}{\gamma_{M1}}$ $V_{wRd} = 2.997\cdot 10^3\,kN$

Alternative 2 = transverse stiffeners $V_{Rd_2} := V_{wRd}$

Weight $Mass_2 := (h_w\cdot t_w\cdot l_w + n_{st}\cdot b_{st}\cdot t_{st}\cdot h_w)\cdot spv$ $Mass_2 = 314\ \text{kg}$

3) Transverse intermediate, rigid stiffeners



Number of transverse stiffeners $n_{st} := 3$ $endpost = 1$

Web panel length $a := \frac{l_w}{n_{st}+1}$ $a = 1 \cdot 10^3 \cdot mm$ $\frac{a}{h_w} = 0.5$

(5.97) $k_\tau := if\left[\frac{a}{h_w} > 1.00, 5.34 + 4.00 \cdot \left(\frac{h_w}{a}\right)^2, 4.00 + 5.34 \cdot \left(\frac{h_w}{a}\right)^2\right]$ $k_\tau = 25.36$

(5.96) $\lambda_w := \frac{0.81}{\sqrt{k_\tau}} \cdot \frac{h_w}{t_w} \cdot \sqrt{\frac{f_{ow}}{E}}$ $\lambda_w = 1.527$

Table 5.12 $\rho_v := if\left(\lambda_w > 0.949, \frac{1.32}{1.66 + \lambda_w}, \frac{0.48}{\lambda_w}\right)$ $\rho_v = 0.414$

Table 5.12 $\rho_v := if(\rho_v > \eta, \eta, \rho_v)$ $\rho_v = 0.414$

(5.95) $V_{wRd} := \rho_v \cdot t_w \cdot h_w \cdot \frac{f_{ow}}{\gamma_{M1}}$ $V_{wRd} = 4.01 \cdot 10^3 \cdot kN$

Check rigidity of stiffener

$t_{st} := 18 \cdot mm$ $b_{st} := 220 \cdot mm$

Figure 5.21 $A_{st} := t_{st} \cdot b_{st} + 30 \cdot t_w^2$ $A_{st} = 1.071 \cdot 10^4 \cdot mm^2$

C. G. $e_{st} := \frac{t_{st} \cdot b_{st}^2}{2 \cdot A_{st}}$ $e_{st} = 40.672 \cdot mm$

Second moment of area $I_{st} := \frac{t_{st} \cdot b_{st}^3}{3} - A_{st} \cdot e_{st}^2$ $I_{st} = 4.617 \cdot 10^7 \cdot mm^4$

(5.104) or (5.105) $I_{limit} := if\left(\frac{a}{h_w} < \sqrt{2}, 1.5 \cdot \frac{h_w^3 \cdot t_w^3}{a^2}, 0.75 \cdot h_w \cdot t_w^3\right)$ $I_{limit} = 4.05 \cdot 10^7 \cdot mm^4$

$I_{st} > I_{limit}$ OK!

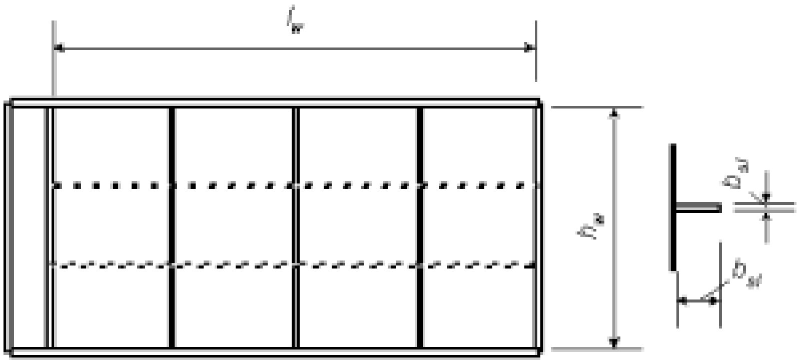
Axial force in stiffener $N_{st} := V_{wRd} - 1.4 \cdot t_w^2 \cdot \sqrt{E \cdot f_{ow}} \cdot \frac{1}{\gamma_{M1}}$

TALAT 2301.06 (6.03h) $\sigma_{st} := \frac{N_{st}}{A_{st}}$ $\sigma_{st} = 241 \cdot MPa$ $\frac{f_{ow}}{\gamma_{M1}} = 323 \cdot MPa$ $\sigma_{st} < f_{ow}$ OK!

Alternative 3 = transverse intermediate, rigid stiffeners $V_{Rd_3} := V_{wRd}$

Weight $Mass_3 := (h_w \cdot t_w \cdot l_w + n_{st} \cdot b_{st} \cdot t_{st} \cdot h_w) \cdot spv$ $Mass_3 = 345$ kg

4) Rigid transverse stiffeners and longitudinal stiffeners



Number of longitudinal stiffeners $n_{sl} := 2$

Longitudinal plate stiffener $t_{sl} := 15 \cdot mm$ $b_{sl} := 120 \cdot mm$ $\frac{b_{sl}}{t_{sl}} = 8$

Number of rigid transverse stiffeners $n_{st} = 3$

If "rigid end post" then *endpost* = 1 else *endpost* = 0 $endpost := 1$

Buckling of longitudinal stiffener

Area $A_{sl} := t_{sl} \cdot b_{sl} + 40 \cdot t_w^2$ $A_{sl} = 1.08 \cdot 10^4 \cdot mm^2$

GC $e_{sl} := \frac{t_{sl} \cdot b_{sl}^2}{2 \cdot A_{sl}}$ $e_{sl} = 10 \cdot mm$

Second moment of area $I_{sl} := \frac{t_{sl} \cdot b_{sl}^3}{3} - A_{sl} \cdot e_{sl}^2$ $I_{sl} = 7.56 \cdot 10^6 \cdot mm^4$

Panel length $a := \frac{l_w}{n_{st} + 1}$ $a = 1 \cdot 10^3 \cdot mm$

(5.99) $k_{\tau st} := 9 \cdot \left(\frac{h_w}{a}\right)^2 \cdot \left(\frac{n_{sl} \cdot I_{sl}}{t_w^3 \cdot h_w}\right)^{\frac{3}{4}}$ $k_{\tau st} = 65.916$

(5.99) $k_{\tau stmin} := \frac{2.1}{t_w} \cdot \left(\frac{n_{sl} \cdot I_{sl}}{h_w}\right)^{\frac{1}{3}}$ $k_{\tau stmin} = 2.748$

(5.99) $k_{\tau st} := if\left(k_{\tau st} < k_{\tau stmin}, k_{\tau stmin}, k_{\tau st}\right)$ $k_{\tau st} = 65.916$

(5.97) $k_{\tau} := 5.34 + 4.00 \cdot \left(\frac{h_w}{a}\right)^2 + k_{\tau st}$ $k_{\tau} = 87.256$

(5.96) $\lambda_w := \frac{0.81}{\sqrt{k_{\tau}}} \cdot \frac{h_w}{t_w} \cdot \sqrt{\frac{f_{ow}}{E}}$ $\lambda_w = 0.823$

Whole panel $\lambda_{wtot} := \lambda_w$ $\lambda_{wtot} = 0.823$

Sub panel

Depth	$h_1 := \frac{h_w}{3}$	$h_1 = 667 \cdot mm$
(5.98)	$k_\tau := 4.00 + 5.34 \cdot \left(\frac{h_1}{a}\right)^2$	$k_\tau = 6.373$
(5.96)	$\lambda_w := \frac{0.81}{\sqrt{k_\tau}} \cdot \frac{h_1}{t_w} \cdot \sqrt{\frac{f_{ow}}{E}}$	$\lambda_w = 0.823$
Sub-panel	$\lambda_{wsub} := \lambda_w$	$\lambda_{wsub} = 1.016$
	compare	$\lambda_{wtot} = 0.823$

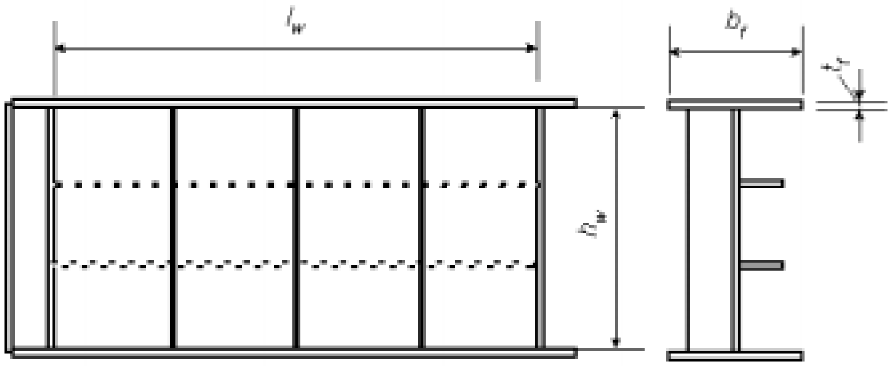
The larger of the slenderness parameter λ_{wtot} for the total panel and λ_{wsub} for the sub panels is used. If $\lambda_{wtot} > \lambda_{wsub}$ then the stiffeners are flexible else the stiffeners are rigid.

	$\lambda_w := if(\lambda_{wtot} > \lambda_{wsub}, \lambda_{wtot}, \lambda_{wsub})$	$\lambda_w = 1.016$
Table 5.12	$\rho_v := if\left(\lambda_w > 0.949, \frac{1.32}{1.66 + \lambda_w}, \frac{0.48}{\lambda_w}\right)$	$\rho_v = 0.493$
Table 5.12	$\rho_v := if(\rho_v > \eta, \eta, \rho_v)$	
Table 5.12	$\rho_v := if\left(endpost = 0, if\left(\rho_v > \frac{0.48}{\lambda_w}, \frac{0.48}{\lambda_w}, \rho_v\right), \rho_v\right)$	$\rho_v = 0.493$
(5.95)	$V_{wRd} := \rho_v \cdot t_w \cdot h_w \cdot \frac{f_{ow}}{\gamma_{M1}}$	$V_{wRd} = 4.78 \cdot 10^3 \cdot kN$

Alternative 4 = transverse and longitudinal intermediate stiffeners $V_{Rd_4} := V_{wRd}$

Weight $Mass_4 := \left(h_w \cdot t_w \cdot l_w + n_{st} \cdot b_{st} \cdot t_{st} \cdot h_w + n_{sl} \cdot t_{sl} \cdot b_{sl} \cdot l_w\right) \cdot spv$ $Mass_4 = 380$ kg

5) Shear resistance contribution of the flanges added to girder 4



The panel is at the end of the plate girder. Then the bending moment is neglected

Flange thickness and width $t_f := 50 \cdot mm$ $b_f := 750 \cdot mm$

Yield strength of flange plate $f_{of} := 355 \cdot MPa$

Design shear force $V_{Ed} := 6000 \cdot kN$

Design bending moment $M_{Ed} := V_{Ed} \cdot l_w$ $M_{Ed} = 2.4 \cdot 10^4 \circ kN \cdot m$

(5.101) $M_{fRd} := b_f \cdot t_f \cdot (h_w + t_f) \cdot \frac{f_{of}}{\gamma_{M1}}$ $\frac{M_{Ed}}{M_{fRd}} = 0.967$ $M_{fRd} = 2.481 \cdot 10^4 \circ kN \cdot m$

$a = 1 \cdot 10^3 \circ mm$

(5.101) $c := \left(0.08 + \frac{4.4 \cdot b_f \cdot t_f^2 \cdot f_{of}}{t_w \cdot h_w^2 \cdot f_{ow}}\right) \cdot a$ $c = 217.5 \circ mm$

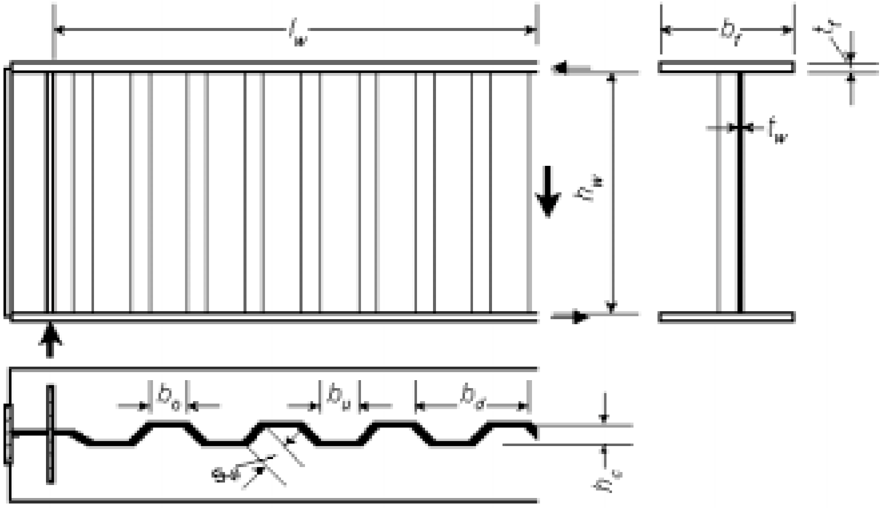
(5.101) $V_{fRd} := if\left[M_{Ed} < M_{fRd}, \frac{b_f \cdot t_f^2 \cdot f_{of}}{c \cdot \gamma_{M1}} \cdot \left[1 - \left(\frac{M_{Ed}}{M_{fRd}}\right)^2\right], 0\right]$ $V_{fRd} = 178.626 \circ kN$

Sum $V_{Rd_5} := V_{wRd} + V_{fRd}$

Example 5 = inclusive shear resistance contribution of the flanges $V_{Rd_5} = 4.955 \cdot 10^3 \circ kN$

Weight $Mass_5 := (h_w \cdot t_w \cdot l_w + n_{st} \cdot b_{st} \cdot t_{st} \cdot h_w + n_{sl} \cdot t_{sl} \cdot b_{sl} \cdot l_w) \cdot spv$ $Mass_5 = 380$ kg

6) Corrugated web



Web depth $h_w := 2000 \cdot mm$ $\quad MPa := 1000000 \cdot Pa$

Web thickness $t_w := 12 \cdot mm$ $\quad kN := 1000 \cdot newton$

Total length of web panel $l_w := 4000 \cdot mm$

0,2 proof strength of web plate material $f_{ow} := 355 \cdot MPa$

Ultimate strength of web plate material $f_{uw} := 470 \cdot MPa$

Elastic modulus $E := 70000 \cdot MPa$

Trapezoidal web: $b_o := 140 \cdot mm$ $\quad b_u := 140 \cdot mm$

$b_d := 400 \cdot mm$ $\quad h_c := 100 \cdot mm$

$s_w := \sqrt{\left[\left(b_d - b_o - b_u\right) \cdot 0.5\right]^2 + h_c^{\,2}}$ $\qquad s_w = 116.619 \circ mm$

Partial coefficient $\gamma_{M1} := 1.1$

Table 5.12 $\quad \eta := 0.4 + 0.2 \cdot \dfrac{f_{uw}}{f_{ow}}$ $\qquad \eta = 0.665$

Local buckling of b_o , b_u and b_u

Max width $\quad b_m := if\left(b_o < b_u, b_u, b_o\right)$ $\qquad b_m = 140 \circ mm$

$b_m := if\left(b_m > s_w, b_m, s_w\right)$ $\qquad b_m = 140 \circ mm$

(5.96) $\quad \lambda_w := 0.35 \cdot \dfrac{b_m}{t_w} \cdot \sqrt{\dfrac{f_{ow}}{E}}$ $\qquad \lambda_w = 0.291$

Table 5.12

Non-rigid end post $\quad \rho_v := if\left(\lambda_w < \dfrac{0.48}{\eta}, \eta, \dfrac{0.48}{\lambda_w}\right)$ $\qquad \eta = 0.665$ $\qquad \rho_v = 0.665$

(5.117) $\quad V_{wRd} := 0.7 \cdot \rho_v \cdot t_w \cdot h_w \cdot \dfrac{f_{ow}}{\gamma_{M1}}$ $\qquad V_{wRd} = 3.604 \cdot 10^3 \circ kN$

Trapezoidal web, width b_d

Area $A := b_o \cdot t_w + b_u \cdot t_w + 2 \cdot s_w \cdot t_w$ $A = 6.159 \cdot 10^3 \, mm^2$

Gravity centre $e_{gc} := \dfrac{b_o \cdot t_w \cdot h_c + 2 \cdot s_w \cdot t_w \cdot 0.5 \cdot h_c}{A}$ $e_{gc} = 50 \, mm$

Second moment of area $I_x := \left(b_o \cdot t_w \cdot h_c^2 + 2 \cdot s_w \cdot t_w \cdot \dfrac{h_c^2}{3} - A \cdot e_{gc}^2 \right) \cdot \dfrac{1}{b_d}$ $I_x = 2.683 \cdot 10^4 \, mm^3$

(5.122) $I_z := \dfrac{b_d}{b_u + b_o + 2 \cdot s_w} \cdot \dfrac{t_w^3}{10.9}$ $I_z = 123.554 \, mm^3$

(5.121) $V_{o.cr} := \dfrac{60 \cdot E}{h_w} \cdot \left(I_z \cdot I_x^3 \right)^{\frac{1}{4}}$ $V_{o.cr} = 1.468 \cdot 10^4 \, kN$

(5.120) $\lambda_{ow} := \sqrt{\dfrac{h_w \cdot t_w \cdot f_{ow}}{V_{o.cr}}}$ $0.7 \cdot \rho_v = 0.465$ $\lambda_{ow} = 0.762$

(5.119) $\chi_o := \dfrac{0.60}{0.8 + \lambda_{ow}^2}$ $\chi_o := if\left(\chi_o > 0.7 \cdot \rho_v , 0.7 \cdot \rho_v , \chi_o \right)$ $\chi_o = 0.435$

(5.118) $V_{o.Rd} := \chi_o \cdot h_w \cdot t_w \cdot \dfrac{f_{ow}}{\gamma_{M1}}$ $V_{o.Rd} = 3.366 \cdot 10^3 \, kN$

Min $V_{Rd_6} := if\left(V_{wRd} > V_{o.Rd} , V_{o.Rd} , V_{wRd} \right)$ $V_{Rd_6} = 3.366 \cdot 10^3 \, kN$

Weight $Mass_6 := \left[t_w \cdot l_w \cdot h_w \cdot \dfrac{\left(b_u + b_o + 2 \cdot s_w \right)}{b_d} \right] \cdot spv$ $Mass_6 = 296$ kg

Summary

1) No intermediate stiffeners $t_{w1\_5} = 15 \circ mm$
2) Transverse flexible stiffeners
3) Transverse rigid stiffeners
4) Transverse rigid + longitudinal stiffeners
5) Transverse rigid + longitudinal stiffeners
   + contribution of flanges
6) Trapezoidal web $t_w = 12 \circ mm$

$i =$	$\frac{V_{Rd_i}}{1000 \cdot kN} =$	Increase in resistance $f_i =$	$\frac{Mass_i}{Mass_1} =$	Weight / resistance $\frac{Mass_i}{Mass_1} \cdot \frac{1}{f_i} =$
1	2.711	1	1	1
2	2.997	1.106	1.09	0.986
3	4.01	1.479	1.198	0.81
4	4.777	1.762	1.318	0.748
5	4.955	1.828	1.318	0.721
6	3.366	1.242	1.026	0.827

"No buckling" $V_{0.Rd} := \eta \cdot h_w \cdot t_{w1\_5} \cdot \frac{f_{ow}}{\gamma_{M1}}$ $V_{0.Rd} = 6.436 \circ 1000 \cdot kN$

Comments

The second column gives the increase in shear resistance compared to girder 1 when adding stiffeners. By adding both transversal and horizontal stiffeners (5) the resistance can be almost doubled.

The last column, "weight per resistance" show that stiffeners give lighter girders, but the comparison does nc pay regard to the cost for welding of the stiffeners.

The contribution of the flanges (girder 5) increase the resistance with $\alpha = 3.7 \circ \%$ compared to girder 4

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