TALAT Lecture 2301

Design of Members

Shear Force

Example 6.8 : Shear force resistance of orthotropic double-skin plate

9 pages

Advanced Level

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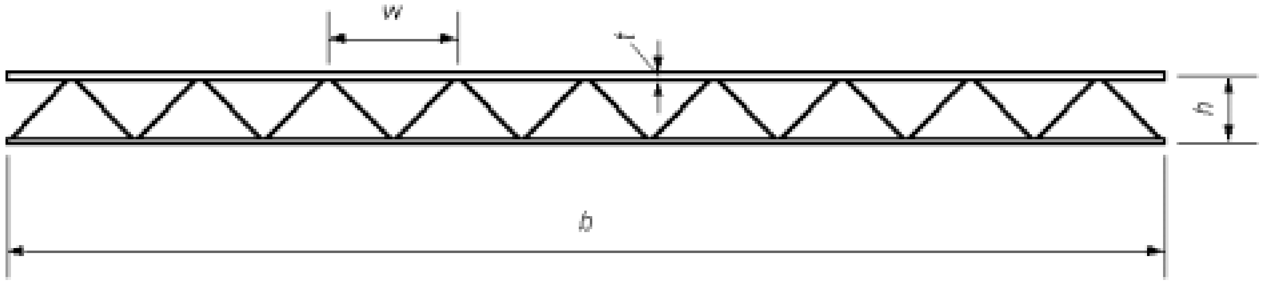
Example 6.8. Shear force resistance of orthotropic double-skin plate



Input (highlighted)

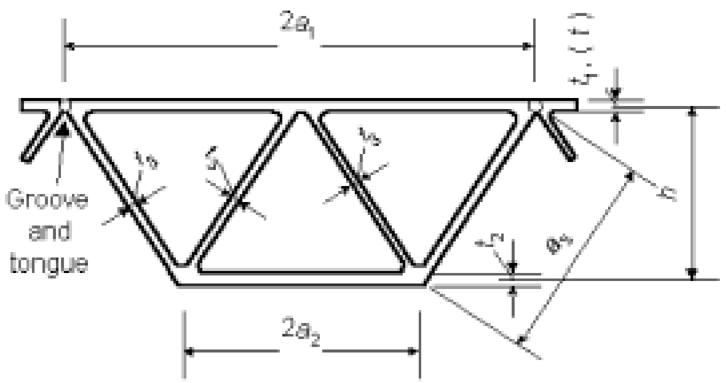
$N \equiv newton$

$kN \equiv 1000 \cdot N$

$MPa \equiv 10^6 \cdot Pa$

Plate thickness	$t := 5 \cdot mm$	$t_1 := t$	$f_o := 240 \cdot MPa$
Plate width	$b := 300000 \cdot mm$		$f_u := 260 \cdot MPa$
Plate length	$L := 5000 \cdot mm$		$E := 70000 \cdot MPa$
"Pitch" (2a or d)	$w := 160 \cdot mm$	$a := \frac{w}{2}$	$\gamma_{M1} := 1.1$
If heat-treated alloy, then ht = 1 else ht = 0		$cf := 0$	$ht := 1$

a) Profiles with groove and tongue



Half bottom flange	$a_2 := 40 \cdot mm$
Thickness of bottom flange	$t_2 := 5 \cdot mm$
Profile depth	$h := 70 \cdot mm$
Web thickness	$t_3 := 5 \cdot mm$
Half width of trapezoidal stiffener at the top	$a_1 := 80 \cdot mm$
Number of webs	$n_w := 4$

Width of web $a_3 := \sqrt{(a_1 - a_2)^2 + h^2}$ $a_3 = 80.6 \circ mm$

Local buckling of top flange

Length and width of web panel: $L = 5\cdot10^{3}\,mm$ $\quad a_m := a_1 \quad a_m = 80\,mm \quad \dfrac{L}{a_m} = 62.5$

(5.97) $k_\tau := if\left[\dfrac{L}{a_m} > 1.00, 5.34 + 4.00\cdot\left(\dfrac{a_m}{L}\right)^2, 4.00 + 5.34\cdot\left(\dfrac{a_m}{L}\right)^2\right]$ $\quad k_\tau = 5.341$

(5.96) $\lambda_w := \dfrac{0.81}{\sqrt{k_\tau}}\cdot\dfrac{a_m}{t_1}\cdot\sqrt{\dfrac{f_o}{E}}$ $\quad \lambda_w = 0.328$

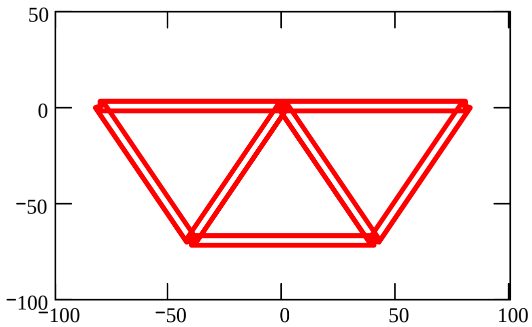
Table 5.12 $\rho_v := \dfrac{0.48}{\lambda_w} \quad \eta := 0.4 + 0.2\cdot\dfrac{f_u}{f_o}$ $\quad \rho_v = 1.462$

$\eta = 0.617$

Table 5.12 $\rho_v := if(\rho_v > \eta, \eta, \rho_v)$ $\quad \rho_v = 0.617$

(5.95), two flanges $V_{w.Rd} := \rho_v\cdot b\cdot(t_1 + t_3)\cdot\dfrac{f_o}{\gamma_{M1}}$ $\quad V_{w.Rd} = 4.036\cdot10^{5}\,kN$

No reduction for local buckling



Overall buckling, shear

Cross sectional area $A := 2\cdot t_1\cdot a + 2\cdot t_2\cdot a_2 + 2\cdot t_3\cdot a_3\cdot\dfrac{n_w}{2}$ $\quad A = 2.812\cdot10^{3}\,mm^{2}$

Gravity centre $e := \dfrac{2\cdot t_2\cdot a_2\cdot h + 2\cdot t_3\cdot a_3\cdot\dfrac{h}{2}\cdot\dfrac{n_w}{2}}{A}$ $\quad e = 30.022\,mm$

Second moment of area $I_L := 2\cdot t_2\cdot a_2\cdot h^2 + 2\cdot t_3\cdot a_3\cdot\dfrac{h^2}{3}\cdot\dfrac{n_w}{2} - A\cdot e^2$ $\quad I_L = 2.059\cdot10^{6}\,mm^{4}$

Approx. torsional constant $I_T := \dfrac{4\cdot\left[h\cdot(a_1 + a_2)\right]^2}{\dfrac{2\cdot a_1}{t_1} + \dfrac{2\cdot a_2}{t_2} + 2\cdot\dfrac{a_3}{t_3}}$ $\quad I_T = 3.517\cdot10^{6}\,mm^{4}$

Rigidities of orthotropic plate

Table 5.10 $B_x := \frac{E \cdot I_L}{2 \cdot a}$ $\nu := 0.3$ $B_x = 9.007 \cdot 10^8 \circ \left(\frac{N \cdot mm^2}{mm}\right)$

Table 5.10 $B_y := 0.001 \cdot N \cdot mm$ $G := \frac{E}{2 \cdot (1 + \nu)}$ $B_y = 1 \cdot 10^{-3} \circ \left(\frac{N \cdot mm^2}{mm}\right)$

Table 5.10 $H := \frac{G \cdot I_T}{2 \cdot a}$ $H = 5.918 \cdot 10^8 \circ \left(\frac{N \cdot mm^2}{mm}\right)$

Elastic buckling load

(5.83) (5.84) $\phi := \frac{L}{b} \cdot \left(\frac{B_y}{B_x}\right)^{\frac{1}{4}}$ $\eta := \frac{H}{\sqrt{B_x \cdot B_y}}$ $\phi = 1.711 \cdot 10^{-5}$

$\eta = 6.236 \cdot 10^5$

(5.82) $k_\tau := 3.25 - 0.567 \cdot \phi + 1.92 \cdot \phi^2 + \left(1.95 + 0.1 \cdot \phi + 2.75 \cdot \phi^2\right) \cdot \eta$ $k_\tau = 1.216 \cdot 10^6$

(5.81) $V_{o.cr} := \frac{k_\tau \cdot \pi^2}{b} \cdot \left(B_x \cdot B_y^3\right)^{\frac{1}{4}}$ $V_{o.cr} = 0.039 \circ kN$

Buckling resistance

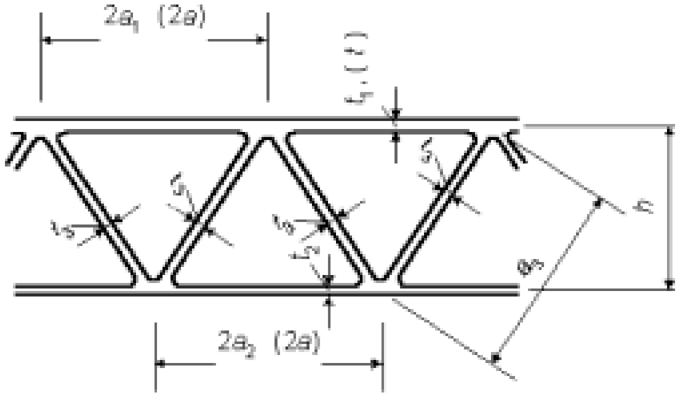
(5.120) $\lambda_{ow} := \sqrt{\frac{b \cdot t_1 \cdot f_o}{V_{o.cr}}}$ $\lambda_{ow} = 3.039 \cdot 10^3$

(5.119) $\chi_o := \frac{0.6}{0.8 + \lambda_{ow}^2}$ $\chi_o := if\left(\chi_o > 0.6, 0.6, \chi_o\right)$ $\chi_o = 6.495 \cdot 10^{-8}$

(5.118) $V_{o.Rd} := \chi_o \cdot b \cdot t_1 \cdot \frac{f_o}{\gamma_{M1}}$ $V_{o.Rd} = 0.021 \circ kN$

$V_{Rd} := if\left(V_{w.Rd} < V_{o.Rd}, V_{w.Rd}, V_{o.Rd}\right)$ $V_{Rd} = 0.021 \circ kN$

b) Truss cross section



Half bottom flange $\quad a_2 := \frac{a}{2} \qquad a_1 := \frac{a}{2} \qquad a = 80 \cdot mm$

Stiffener depth $\quad h := 70 \cdot mm \qquad a_1 = 40 \cdot mm$

Thickness of bottom flange $\quad t_2 := 5 \cdot mm \qquad a_2 = 40 \cdot mm$

Web thickness $\quad t_3 := 5 \cdot mm$

Width of web $\quad a_3 := \sqrt{a_1^2 + h^2} \qquad a_3 = 80.623 \cdot mm$

Local buckling of top flange

Length and width of web panel $\quad L = 5 \cdot 10^3 \cdot mm \qquad a_m := 2 \cdot a_1 \qquad a_m = 80 \cdot mm \qquad \frac{L}{a_m} = 62.5$

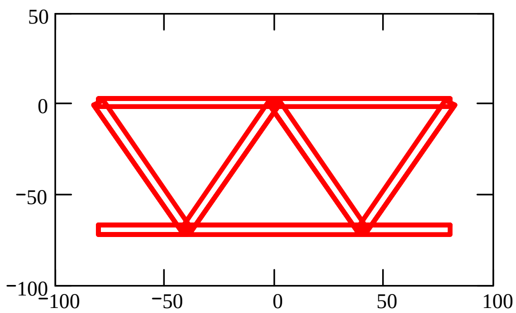
(5.97) $\quad k_\tau := if\left[\frac{L}{a_m} > 1.00, 5.34 + 4.00 \cdot \left(\frac{a_m}{L}\right)^2, 4.00 + 5.34 \cdot \left(\frac{a_m}{L}\right)^2\right] \qquad k_\tau = 5.341$

(5.96) $\quad \lambda_w := \frac{0.81}{\sqrt{k_\tau}} \cdot \frac{a_m}{t_1} \cdot \sqrt{\frac{f_o}{E}} \qquad \lambda_w = 0.328$

Table 5.12 $\quad \rho_v := \frac{0.48}{\lambda_w} \qquad \eta := 0.4 + 0.2 \cdot \frac{f_u}{f_o} \qquad \rho_v = 1.462 \qquad \eta = 0.617$

Table 5.12 $\quad \rho_v := if(\rho_v > \eta, \eta, \rho_v) \qquad \rho_v = 0.617$

(5.95), three flanges (web) $\quad V_{w.Rd} := \rho_v \cdot b \cdot (t_1 + t_2 + t_3) \cdot \frac{f_o}{\gamma_{M1}} \qquad V_{w.Rd} = 6.055 \cdot 10^5 \cdot kN$



Overall buckling, shear

Cross sectional area $A := 2\cdot t_1\cdot a_1 + 2\cdot t_2\cdot a_2 + 2\cdot t_3\cdot a_3$ $\quad A = 1.606\cdot 10^3 \circ mm^2$

Gravity centre $e := \dfrac{2\cdot t_2\cdot a_2\cdot h + 2\cdot t_3\cdot a_3\cdot \dfrac{h}{2}}{A}$ $\quad e = 35\circ mm$

Second moment of area $I_L := 2\cdot t_2\cdot a_2\cdot h^2 + 2\cdot t_3\cdot a_3\cdot \dfrac{h^2}{3} - A\cdot e^2$ $\quad I_L = 1.309\cdot 10^6\circ mm^4$

Torsion constant $I_T := \dfrac{4\cdot\left[h\cdot\left(a_1 + a_2\right)\right]^2}{\dfrac{2\cdot a_1}{t_1} + \dfrac{2\cdot a_2}{t_2} + 2\cdot\dfrac{a_3}{t_3}}$ $\quad I_T = 1.952\cdot 10^6\circ mm^4$

Orthotropic plate constant

Table 5.10 $B_x := \dfrac{E\cdot I_L}{2\cdot a}$ $\quad B_x = 5.728\cdot 10^8\circ\left(\dfrac{N\cdot mm^2}{mm}\right)$

Table 5.10 $B_y := \dfrac{E\cdot t_1\cdot t_2\cdot h^2}{t_1 + t_2}$ $\quad B_y = 8.575\cdot 10^8\circ\left(\dfrac{N\cdot mm^2}{mm}\right)$

Table 5.10 $H := \dfrac{G\cdot I_T}{2\cdot a}$ $\quad H = 3.285\cdot 10^8\circ\left(\dfrac{N\cdot mm^2}{mm}\right)$

Elastic buckling load

(5.83) (5.84) $\phi := \dfrac{L}{b}\cdot\left(\dfrac{B_y}{B_x}\right)^{\frac{1}{4}}$ $\quad \eta := \dfrac{H}{\sqrt{B_x\cdot B_y}}$ $\quad \phi = 0.018$

$\eta = 0.469$

(5.82) $k_\tau := 3.25 - 0.567\cdot\phi + 1.92\cdot\phi^2 + \left(1.95 + 0.1\cdot\phi + 2.75\cdot\phi^2\right)\cdot\eta$ $\quad k_\tau = 4.156$

(5.81) $V_{o.cr} := \dfrac{k_\tau\cdot\pi^2}{b}\cdot\left(B_x\cdot B_y^3\right)^{\frac{1}{4}}$ $\quad V_{o.cr} = 105.983\circ kN$

Buckling resistance

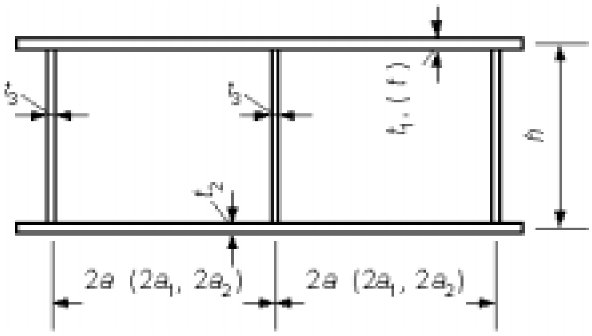
(5.120) $\lambda_{ow} := \sqrt{\dfrac{b\cdot t_1\cdot f_o}{V_{o.cr}}}$ $\quad \lambda_{ow} = 58.282$

(5.119) $\chi_o := \dfrac{0.6}{0.8 + \lambda_{ow}^2}$ $\quad \chi_o := if\left(\chi_o > 0.6, 0.6, \chi_o\right)$ $\quad \chi_o = 1.766\cdot 10^{-4}$

(5.118) $V_{o.Rd} := \chi_o\cdot b\cdot t_1\cdot\dfrac{f_o}{\gamma_{M1}}$ $\quad V_{o.Rd} = 57.795\circ kN$

$V_{Rd} := if\left(V_{w.Rd} < V_{o.Rd}, V_{w.Rd}, V_{o.Rd}\right)$ $\quad V_{Rd} = 57.795\circ kN$

c) Frame cross section



Half bottom flange	$a := 37.5 \cdot mm$	$a_1 := a$	
Stiffener depth	$h := 70 \cdot mm$	$a_2 := a$	
Thickness of top flange	$t_1 := 5 \cdot mm$		$a_1 = 37.5 \, mm$
Thickness of bottom flange	$t_2 := 5 \cdot mm$		$a_2 = 37.5 \, mm$
Web thickness	$t_3 := 5 \cdot mm$		
Width of web	$a_3 := h$		$a_3 = 70 \, mm$

Local buckling of top flange

Thickness $\quad t_m := if(t_1 < t_2, t_1, t_2) \qquad t_m = 5 \, mm$

Length and width of web panel $\quad L = 5 \cdot 10^3 \, mm \qquad a_m := 2 \cdot a_1 \qquad a_m = 75 \, mm \qquad \dfrac{L}{a_m} = 66.667$

(5.97) $$k_\tau := if\left[\frac{L}{a_m} > 1.00, 5.34 + 4.00 \cdot \left(\frac{a_m}{L}\right)^2, 4.00 + 5.34 \cdot \left(\frac{a_m}{L}\right)^2\right] \qquad k_\tau = 5.341$$

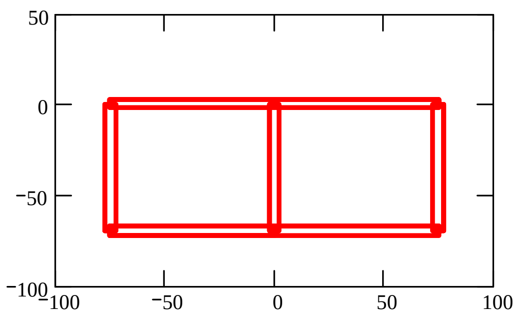
(5.96) $$\lambda_w := \frac{0.81}{\sqrt{k_\tau}} \cdot \frac{a_m}{t_m} \cdot \sqrt{\frac{f_o}{E}} \qquad \lambda_w = 0.308$$

Table 5.12 $$\rho_v := \frac{0.48}{\lambda_w} \qquad \eta := 0.4 + 0.2 \cdot \frac{f_u}{f_o} \qquad \rho_v = 1.559 \qquad \eta = 0.617$$

Table 5.12 $$\rho_v := if(\rho_v > \eta, \eta, \rho_v) \qquad \rho_v = 0.617$$

(5.95), two flanges $$V_{w.Rd} := \rho_v \cdot b \cdot (t_1 + t_2) \cdot \frac{f_o}{\gamma_{M1}} \qquad V_{w.Rd} = 4.036 \cdot 10^5 \, kN$$

No reduction for local buckling



Overall buckling, shear

Cross sectional area $A := 2 \cdot t_1 \cdot a + 2 \cdot t_2 \cdot a_2 + t_3 \cdot a_3$ $\qquad A = 1.1 \cdot 10^3 \circ mm^2$

Gravity centre $e := \dfrac{2 \cdot t_2 \cdot a_2 \cdot h + t_3 \cdot a_3 \cdot \frac{h}{2}}{A}$ $\qquad e = 35 \circ mm$

Second moment of area $I_L := 2 \cdot t_2 \cdot a_2 \cdot h^2 + t_3 \cdot a_3 \cdot \dfrac{h^2}{3} - A \cdot e^2$ $\qquad I_L = 1.062 \cdot 10^6 \circ mm^4$

Torsion constant $I_T := \dfrac{4 \cdot \left[h \cdot \left(a_1 + a_2 \right) \right]^2}{\dfrac{2 \cdot a_1}{t_1} + \dfrac{2 \cdot a_2}{t_2} + 2 \cdot \dfrac{a_3}{t_3}}$ $\qquad I_T = 1.901 \cdot 10^6 \circ mm^4$

Orthotropic plate constant

(5.80d) $B_x := \dfrac{E \cdot I_L}{2 \cdot a}$ $\qquad B_x = 9.909 \cdot 10^8 \circ \left(\dfrac{N \cdot mm^2}{mm} \right)$

(5.80a $B_y := \dfrac{E \cdot t_1^{\,3}}{12 \cdot \left(1 - \nu^2\right)} \cdot \dfrac{10 \cdot b^2}{32 \cdot a^2} \cdot \dfrac{a \cdot t_3^{\,3} + \dfrac{a \cdot t_2^{\,3} \cdot t_3^{\,3}}{t_1^{\,3}} + 6 \cdot h \cdot t_2^{\,3}}{a \cdot t_3^{\,3} + 2 \cdot h \cdot \left(t_1^{\,3} + t_2^{\,3} \right) + \dfrac{3 \cdot h^2 \cdot t_1^{\,3} \cdot t_2^{\,3}}{a \cdot t_3^{\,3}}} \cdot \dfrac{t_1^{\,2}}{L^2}$ $\qquad B_y = 1.118 \cdot 10^7 \circ \left(\dfrac{N \cdot mm^2}{mm} \right)$

(5.80b) $H := \dfrac{2 \cdot E}{3 \cdot \left(1 - \dfrac{t_3}{2 \cdot a}\right)} \cdot \left[\dfrac{t_1^{\,3}}{1 + \dfrac{6 \cdot t_1}{2 \cdot a - t_3}} + \dfrac{t_2^{\,3}}{1 + \dfrac{6 \cdot t_2}{2 \cdot a - t_3}} \right]$ $\qquad H = 8.75 \cdot 10^6 \circ \left(\dfrac{N \cdot mm^2}{mm} \right)$

Elastic buckling load

(5.83) (5.84) $\phi := \dfrac{L}{b} \cdot \left(\dfrac{B_y}{B_x} \right)^{\frac{1}{4}}$ $\qquad \eta := \dfrac{H}{\sqrt{B_x \cdot B_y}}$ $\qquad \phi = 5.432 \cdot 10^{-3}$

$\eta = 0.083$

(5.82) $k_\tau := 3.25 - 0.567 \cdot \phi + 1.92 \cdot \phi^2 + \left(1.95 + 0.1 \cdot \phi + 2.75 \cdot \phi^2 \right) \cdot \eta$ $\qquad k_\tau = 3.409$

(5.81) $V_{o.cr} := \dfrac{k_\tau \cdot \pi^2}{b} \cdot \left(B_x \cdot B_y^{\,3} \right)^{\frac{1}{4}}$ $\qquad V_{o.cr} = 3.848 \circ kN$

Buckling resistance

(5.120) $\lambda_{ow} := \sqrt{\dfrac{b \cdot t_1 \cdot f_o}{V_{o.cr}}}$ $\qquad \lambda_{ow} = 305.887$

(5.119) $\chi_o := \dfrac{0.6}{0.8 + \lambda_{ow}^{\,2}}$ $\qquad \chi_o := if\left(\chi_o > 0.6, 0.6, \chi_o \right)$ $\qquad \chi_o = 6.412 \cdot 10^{-6}$

(5.118) $V_{o.Rd} := \chi_o \cdot b \cdot t_1 \cdot \dfrac{f_o}{\gamma_{M1}}$ $\qquad V_{o.Rd} = 2.099 \circ kN$

$V_{Rd} := if\left(V_{w.Rd} < V_{o.Rd}, V_{w.Rd}, V_{o.Rd} \right)$ $\qquad V_{Rd} = 2.099 \circ kN$

Summary

a)	$V_{Rd.a} = 0 \cdot kN$	$A_a = 2.812 \cdot 10^3 \cdot mm^2$	$\frac{V_{Rd.a}}{A_a} = 7.6 \cdot 10^{-3} \cdot \frac{N}{mm^2}$
b)	$V_{Rd.b} = 57.8 \cdot kN$	$A_b = 3.212 \cdot 10^3 \cdot mm^2$	$\frac{V_{Rd.b}}{A_b} = 18 \cdot \frac{N}{mm^2}$
c)	$V_{Rd.c} = 2.1 \cdot kN$	$A_c = 2.2 \cdot 10^3 \cdot mm^2$	$\frac{V_{Rd.c}}{A_c} = 1 \cdot \frac{N}{mm^2}$