TALAT Lecture 2301

Design of Members

Axial force and bending moment

Example 9.1 : Tension force and bending moment

6 pages

Advanced Level

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Example 9.1. Tension force and bending moment

Dimensions and material properties

Flange height: $h := 200 \cdot mm$

Flange depth: $b := 140 \cdot mm$

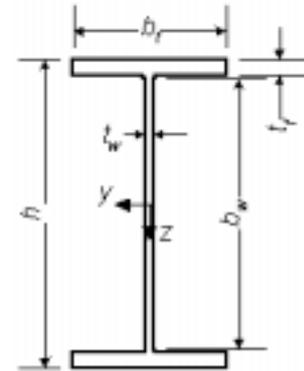Web thickness: $t_w := 12 \cdot mm$

Flange thickness: $t_f := 6 \cdot mm$

Length: $L := 2 \cdot m$

Width of web plate:

$b_w := h - 2 \cdot t_f \qquad b_w = 188 \circ mm$

[1] Table 3.2b Alloy: **EN AW-6082 T6** EP/O t > 5 mm

$$f_{0.2} := 260 \cdot \frac{newton}{mm^2} \qquad f_u := 310 \cdot \frac{newton}{mm^2}$$

[1] (5.4), (5.5) $f_o := f_{0.2} \qquad\qquad f_a := f_u$

[1] (5.6) $\qquad f_v := \dfrac{f_o}{\sqrt{3}} \qquad\qquad f_v = 150 \circ MPa$

$E := 70000 \cdot MPa \qquad G := 27000 \cdot MPa$
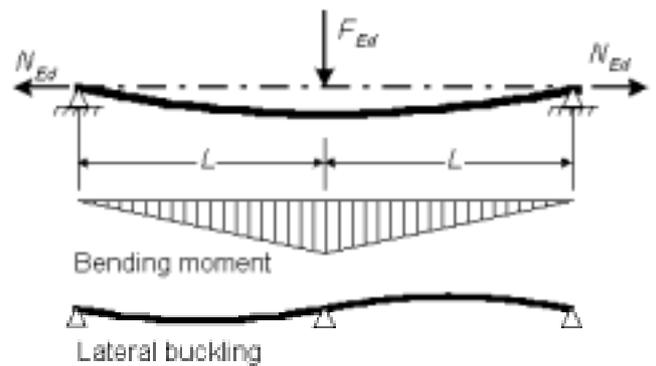
Partial safety factors: $\qquad \gamma_{M1} \bar{=} 1.10 \qquad\qquad \gamma_{M2} \equiv 1.25$

Moment and force

Axial tension force and excentricity $\qquad N_{Ed} := 140 \cdot kN \qquad exc := 114.286 \cdot mm$

Bending moment $\qquad M_{y.Ed} := N_{Ed} \cdot exc \qquad M_{y.Ed} = 16 \circ kNm$

Web-flange corner radius $\qquad r := 4 \cdot mm$

S.I. units $\qquad kN \equiv 1000 \cdot newton \qquad kNm \equiv kN \cdot m \qquad MPa \equiv 1000000 \cdot Pa$

Classification of the cross section in bending. Effective thickness

Web $\qquad \beta_w := 0.40 \cdot \dfrac{b_w}{t_w} \qquad \beta_w = 6.267 \qquad \varepsilon := \sqrt{\dfrac{250 \cdot MPa}{f_o}} \qquad\qquad \varepsilon = 0.981$

[1] Tab. 5.1 $\qquad \beta_{1w} = 9 \cdot \varepsilon \qquad\qquad \beta_{2w} = 13 \cdot \varepsilon \qquad\qquad \beta_{3w} = 18 \cdot \varepsilon$

[1] 5.4.5 $\qquad class_w := if\left(\beta_w \gtrless \beta_{1w}, if\left(\beta_w > \beta_{2w}, if\left(\beta_w > \beta_{3w}, 4, 3\right), 2\right), 1\right) \qquad class_w = 1$

Local buckling: $\rho_{cw} := if\left[\dfrac{\beta_w}{\varepsilon} \leq 18, 1.0, \dfrac{29}{\left(\dfrac{\beta_w}{\varepsilon}\right)} - \dfrac{198}{\left(\dfrac{\beta_w}{\varepsilon}\right)^2}\right] \qquad t_{w.ef} := if\left(class_w \geq 4, t_w \cdot \rho_{cw}, t_w\right) \qquad \rho_{cw} = 1$

$\qquad\qquad t_{w.ef} = 12.0 \circ mm$

Flanges

[1] 5.4.3

$$\beta_f := \frac{b - t_w}{t_f} \qquad \beta_{1f} := 2.5 \cdot \varepsilon \qquad \beta_{2f} := 4 \cdot \varepsilon \qquad \beta_{3f} := 5 \cdot \varepsilon \qquad \beta_f = 21.333$$

[1] Tab. 5.1 $\quad class_f := if\left(\beta_f > \beta_{1f}, if\left(\beta_f > \beta_{2f}, if\left(\beta_f > \beta_{3f}, 4, 3\right), 2\right), 1\right) \qquad\qquad class_f = 4$

[1] 5.4.5

Local buckling: $\rho_{cf} := if\left[\dfrac{\beta_f}{\varepsilon} \le 18, 1.0, \dfrac{29}{\left(\dfrac{\beta_f}{\varepsilon}\right)} - \dfrac{198}{\left(\dfrac{\beta_f}{\varepsilon}\right)^2}\right] \qquad\qquad \rho_{cf} = 0.915$

$$t_{f.ef} := if\left(class_f \ge 4, t_f \rho_{cf}, t_f\right) \qquad\qquad t_{f.ef} = 5.49 \cdot mm$$

Classification of the total cross-section:

$$class := if\left(class_f > class_w, class_f, class_w\right) \qquad\qquad class = 4$$

Cross weld

[1] Tab. 5.2   HAZ softening factor $\rho_{haz} := 0.65$

             Effective thickness, flange $t_{f.haz} := \rho_{haz} \cdot t_f$ $t_{f.haz} = 3.9 \cdot mm$

$$t_{f.ef} := if\left(t_{f.ef} < t_{f.haz}, t_{f.ef}, t_{f.haz}\right) \qquad\qquad t_{f.ef} = 3.9 \cdot mm$$

             Effective thickness, web $t_{w.haz} := \rho_{haz} \cdot t_f$ $t_{w.haz} = 3.9 \cdot mm$

$$t_{w.ef} := if\left(t_{w.ef} < t_{w.haz}, t_{w.ef}, t_{w.haz}\right) \qquad\qquad t_{w.ef} = 3.9 \cdot mm$$

[1] Fig. 5.6   Extent of HAZ i web   (MIG-weld) $t_1 := 0.5 \cdot \left(t_w + t_f\right)$ $t_1 = 9 \cdot mm$

$$b_{haz} := if\left(t_1 > 6 \cdot mm, if\left(t_1 > 12 \cdot mm, if\left(t_1 > 25 \cdot mm, 40 \cdot mm, 35 \cdot mm\right), 30 \cdot mm\right), 20 \cdot mm\right) \qquad b_{haz} = 30 \cdot mm$$

Bending moment resistance

[1] 5.6.2   Elastic modulus of gross cross section W_{el}:

$$A_{gr} := 2 \cdot b \cdot t_f + \left(h - 2 t_f\right) \cdot t_w \qquad\qquad A_{gr} = 3.936 \cdot 10^3 \cdot mm^2$$

$$I_{gr} := \frac{1}{12} \cdot \left[b \cdot h^3 - \left(b - t_w\right) \cdot \left(h - 2 t_f\right)^3\right] \qquad I_{gr} = 2.246 \cdot 10^7 \cdot mm^4$$

$$W_{el} := \frac{I_{gr} \cdot 2}{h} \qquad\qquad W_{el} = 2.246 \cdot 10^5 \cdot mm^3$$

Plastic modulus

$$W_{ple} := \frac{1}{4} \cdot \left[b \cdot h^2 - \left(b - t_w\right) \cdot \left(h - 2 t_f\right)^2\right] \qquad W_{ple} = 2.69 \cdot 10^5 \cdot mm^3$$

Elastic modulus of the effective cross section W_{effe}:

$$t_f = 6 \cdot mm \qquad t_{f.ef} = 3.9 \cdot mm \qquad t_w = 12 \cdot mm \qquad t_{w.ef} = 3.9 \cdot mm$$
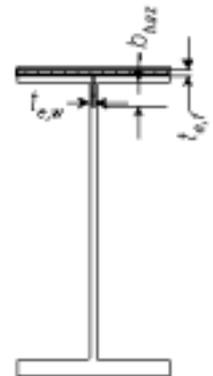
Allowing for local buckling and HAZ:

$$A_{effe} := A_{gr} - b \cdot \left(t_f - t_{f.ef}\right) - b_{haz} \cdot \left(t_w - t_{w.ef}\right) \qquad\qquad A_{effe} = 3.399 \cdot 10^3 \cdot mm^2$$

Shift of gravity centre:

$$e_{ef} := \left[b \cdot (t_f - t_{f.ef}) \cdot \left(\frac{h}{2} - \frac{t_f}{2} \right) + b_{haz} \cdot (t_w - t_{w.ef}) \cdot \left(\frac{h}{2} - t_f - \frac{b_{haz}}{2} \right) \right] \cdot \frac{1}{A_{effe}} \qquad e_{ef} = 14.038 \circ mm$$

Second moment of area with respect to centre of gross cross section:

$$I_{effe} := I_{gr} - b \cdot (t_f - t_{f.ef}) \cdot \left(\frac{h}{2} - \frac{t_f}{2} \right)^2 - \frac{b_{haz}^3}{12} \cdot (t_w - t_{w.ef}) - b_{haz} \cdot (t_w - t_{w.ef}) \cdot \left(\frac{h}{2} - t_f - \frac{b_{haz}}{2} \right)^2$$

$$I_{effe} = 1.816 \cdot 10^7 \circ mm^4$$

Second moment of area with respect to centre of effective cross section:

$$I_{effe} := I_{effe} - e_{ef}^2 \cdot A_{effe} \qquad\qquad I_{effe} = 1.749 \cdot 10^7 \circ mm^4$$

$$W_{effe} := \frac{I_{effe}}{\frac{h}{2} + e_{ef}} \qquad\qquad W_{effe} = 1.533 \cdot 10^5 \circ mm^3$$

[1] Tab. 5.3   Shape factor α for welded, class 4 cross-section $\alpha := \dfrac{W_{effe}}{W_{el}}$ $\alpha = 0.683$

Design moment and axial force resistance of the cross section ($M_{y,Rd}$ and N_{Rd})

[1] (5.14) $M_{y.Rd} := \dfrac{f_o \cdot \alpha \cdot W_{el}}{\gamma_{M1}}$ $N_{Rd} := \dfrac{f_o \cdot A_{effe}}{\gamma_{M1}}$ $M_{y.Rd} = 36.2 \circ kNm$ $N_{Rd} = 803.4 \circ kN$

Section check

[1] (5.42a-c)   Class 4 cross section: $\eta_0 := 1$ $\gamma_0 := 1$ $\xi_0 := 1$

[1] (5.40) $\left(\dfrac{N_{Ed}}{N_{Rd}} \right)^{\eta_0} + \left(\dfrac{M_{y.Ed}}{M_{y.Rd}} \right)^{\gamma_0} = 0.616$

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Lateral-torsional buckling

[1] 5.9.4.3

Lateral stiffness constant $\qquad I_z := \dfrac{2 \cdot b^3 \cdot t_f}{12} + \dfrac{h \cdot t_w^{\,3}}{12}$ $\qquad\qquad I_z = 2.773 \cdot 10^6 \circ mm^4$

[1] Figure J.2 Varping constant: $\qquad I_w := \dfrac{\left(h - t_f\right)^2 \cdot I_z}{4}$ $\qquad\qquad I_w = 2.609 \cdot 10^{10} \circ mm^6$

Torsional constant: $\qquad I_t := \dfrac{2 \cdot b \cdot t_f^{\,3} + h \cdot t_w^{\,3}}{3}$ $\qquad\qquad I_t = 1.354 \cdot 10^5 \circ mm^4$

Length $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad L = 2 \circ m$

[1] H.1.2 Moment relation $\qquad\qquad \psi := 1$ $\qquad\qquad\qquad\qquad \psi = 1$

[1] H.1.2(6) C_1 - constant $\qquad\qquad C_1 := 1.88 - 1.4 \cdot \psi + 0.52 \cdot \psi^2$ $\qquad C_1 = 1$

Shear modulus $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad G = 2.7 \cdot 10^4 \circ MPa$

[1] H.1.3(3) $M_{cr} := \dfrac{C_1 \cdot \pi^2 \cdot E \cdot I_z}{L^2} \cdot \sqrt{\dfrac{I_w}{I_z} + \dfrac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z}}$ $\qquad W_y := W_{el}$ $\qquad M_{cr} = 62.517 \circ kN \cdot m$

[1] 5.6.6.3(3) $\lambda_{LT} := \sqrt{\dfrac{\alpha_{\cdot} W_y \cdot f_o}{M_{cr}}}$ $\qquad\qquad\qquad\qquad\qquad \lambda_{LT} = 0.799$

[1] 5.6.6.3(2) $\alpha_{LT} := if(class > 2, 0.2, 0.1)$ $\qquad\qquad\qquad\qquad \alpha_{LT} = 0.2$

$\lambda_{0LT} := if(class > 2, 0.4, 0.6)$ $\qquad\qquad\qquad\qquad \lambda_{0LT} = 0.4$

[1] 5.6.6.3(1) $\phi_{LT} := 0.5 \cdot \left[1 + \alpha_{LT} \cdot \left(\lambda_{LT} - \lambda_{0LT}\right) + \lambda_{LT}^2\right]$ $\qquad \phi_{LT} = 0.859$

$\chi_{LT} := \dfrac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{LT}^2}}$ $\qquad\qquad\qquad\qquad \chi_{LT} = 0.851$

Design moment and axial force resistance of the cross section (no HAZ)

[1] (5.14) $M_{c.Rd} := \chi_{LT} \dfrac{f_o \cdot W_{el}}{\gamma_{M1}}$ $\qquad N_{Rd} := \dfrac{f_o \cdot A_{gr}}{\gamma_{M1}}$ $\qquad M_{c.Rd} = 45.2 \circ kNm \qquad N_{Rd} = 930.3 \circ kN$

Lateral-torsional buckling check

[1] 5.9.3.1 $\quad \psi_{vec} := 0.8 \qquad W_{com} := W_{el}$

$A_{gr} = 3.936 \cdot 10^3 \circ mm^2$

[1] (5.38) $\quad \sigma_{com.Ed} = \dfrac{M_{y.Ed}}{W_{com}} - \psi_{vec} \cdot \dfrac{N_{Ed}}{A_{gr}}$

$N_{Ed} = 140 \circ kN$

$\qquad\qquad \sigma_{com.Ed} = 42.8 \circ MPa$

$M_{y.Ed} = 16 \circ kNm$

[1] (5.39) $\quad M_{eff.Ed} := W_{com} \cdot \sigma_{com.Ed}$

$M_{eff.Ed} = 9.61 \circ kNm$

$M_{c.Rd} = 45.196 \circ kNm$

$\qquad\qquad\qquad M_{eff.Ed} < M_{c.Rd} \qquad$ OK!